3-PRR parallel chain manipulator

Project Report

MAE 593

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Introduction:

The 3PRR parallel chain manipulator has an analytical solution.

$$X_{EE} = D_i \cos\theta_{1i} + L_{1i}\cos\theta_{2i} + L_{2i}\cos\theta_{3i} + X_{Bi}\ (i = 1,2,3)$$

$$Y_{EE} = D_i \sin\theta_{1i} + L_{1i}\sin\theta_{2i} + L_{2i}\sin\theta_{3i} + Y_{Bi}\ (i = 1,2,3)$$

$$\emptyset_{EE} = \theta_{3i} + \delta_{3i} (i = 1,2,3)$$

Thus for given $(X_{EE}, Y_{EE}, \emptyset_{EE})$ we can solve for (D_i, θ_{2i}) in case of inverse kinematics and vice versa for forward kinematics.

$$D_i \cos\theta_{1i} + L_{1i}\cos\theta_{2i} = X_{EE} - X_{Bi} - L_{2i}\cos\theta_{3i}$$

$$D_i \sin\theta_{1i} + L_{1i}\sin\theta_{2i} = Y_{EE} - Y_{Bi} - L_{2i}\sin\theta_{3i}$$

Differentiating with respect to time gives

$$\dot{D}_i \cos\theta_{1i} - L_{1i}\sin\theta_{2i}\,\dot{\theta}_{2i} = \dot{X}_{EE} + L_{2i}\sin\theta_{3i}\,\dot{\theta}_{3i} (i = 1,2,3)$$

$$\dot{D}_i \sin\theta_{1i} + L_{1i}\cos\theta_{2i}\,\dot{\theta}_{2i} = \dot{Y}_{EE} - L_{2i}\cos\theta_{3i}\,\dot{\theta}_{3i} (i = 1,2,3)$$

$$\dot{\theta}_{3i} = \dot{\emptyset}_{EE} (i = 1,2,3)$$

Thus we have,

$$A * \begin{pmatrix} \dot{D}_1 \\ \dot{D}_2 \\ \dot{D}_3 \\ \dot{\theta}_{21} \\ \dot{\theta}_{22} \\ \dot{\theta}_{23} \end{pmatrix} + B * \begin{pmatrix} \dot{X}_{EE} \\ \dot{Y}_{EE} \\ \dot{\emptyset}_{EE} \end{pmatrix} = 0$$

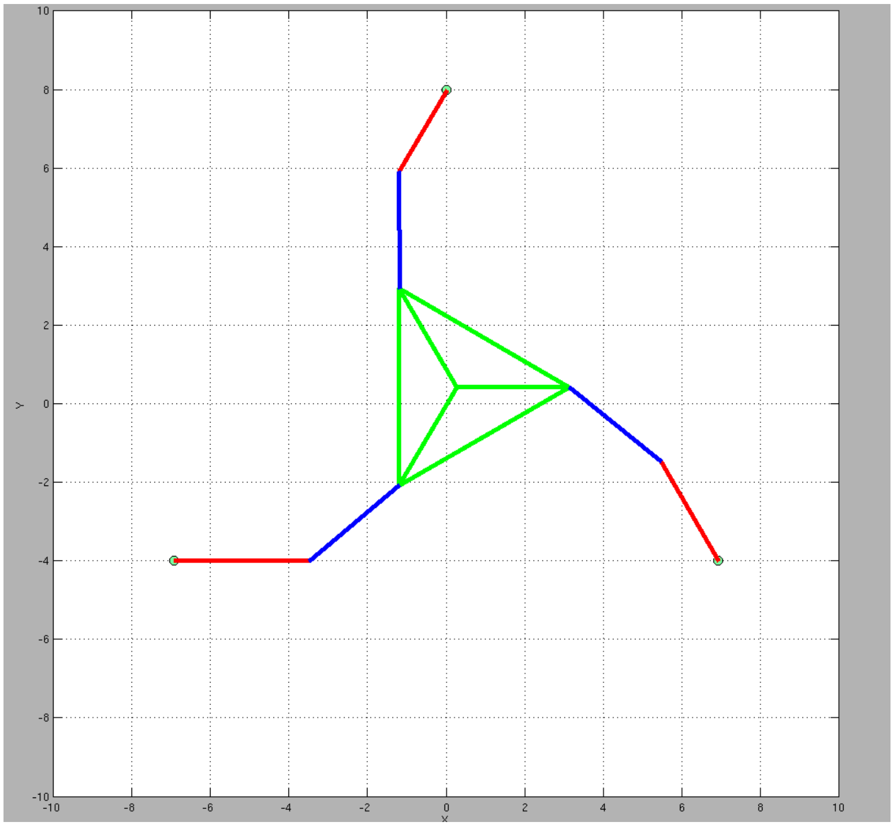
$$where\ A = \begin{pmatrix} 1 & 0 & -L_{21}\sin\theta_{31} \\ 1 & 0 & -L_{22}\sin\theta_{32} \\ 1 & 0 & -L_{23}\sin\theta_{33} \\ 0 & 1 & L_{21}\cos\theta_{31} \\ 0 & 1 & L_{22}\cos\theta_{32} \\ 0 & 1 & L_{23}\cos\theta_{33} \end{pmatrix}$$

$$and\ B =$$

$$\begin{pmatrix} \cos\theta_{11} & 0 & 0 & -L_{11}\sin\theta_{21} & 0 & 0 \\ 0 & \cos\theta_{12} & 0 & 0 & -L_{12}\sin\theta_{22} & 0 \\ 0 & 0 & \cos\theta_{13} & 0 & 0 & -L_{13}\sin\theta_{23} \\ \sin\theta_{11} & 0 & 0 & L_{11}\cos\theta_{21} & 0 & 0 \\ 0 & \sin\theta_{12} & 0 & 0 & L_{12}\cos\theta_{22} & 0 \\ 0 & 0 & \sin\theta_{13} & 0 & 0 & L_{13}\cos\theta_{23} \end{pmatrix}$$

$$and\ J = -inverse(B) * A$$

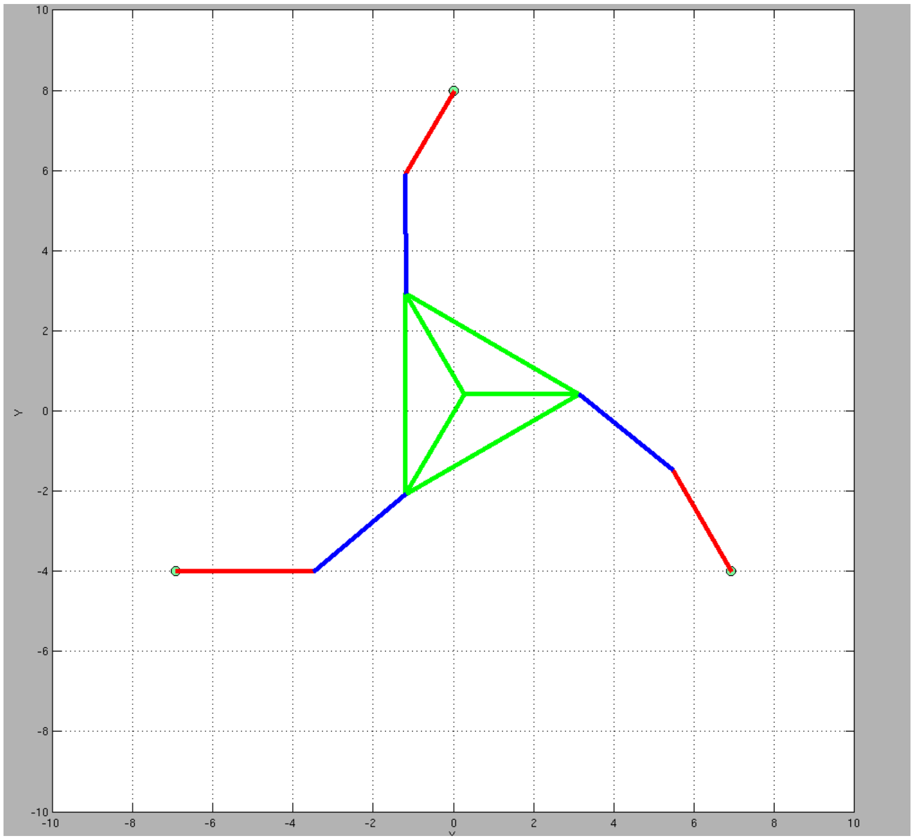
Forward Kinematics



$$(D_1, D_2, D_3) = (3.44, 2.93, 2.38), (\theta_{11}, \theta_{12}, \theta_{13}) = (40.11, 140.83, 270)$$

Please see drop box folder for code.

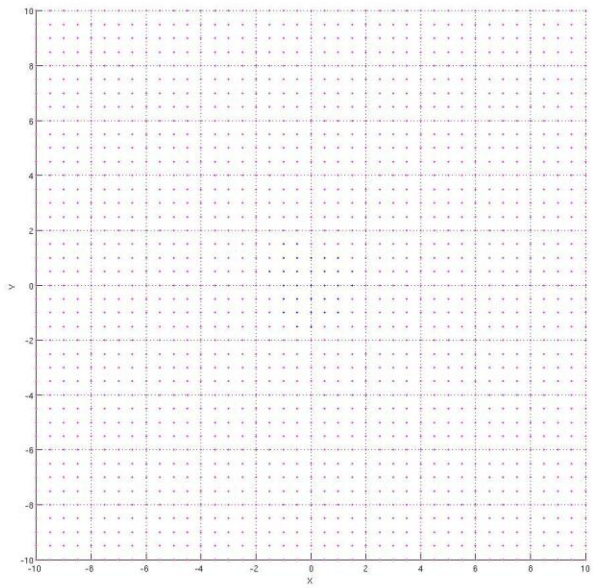
Inverse Kinematics:

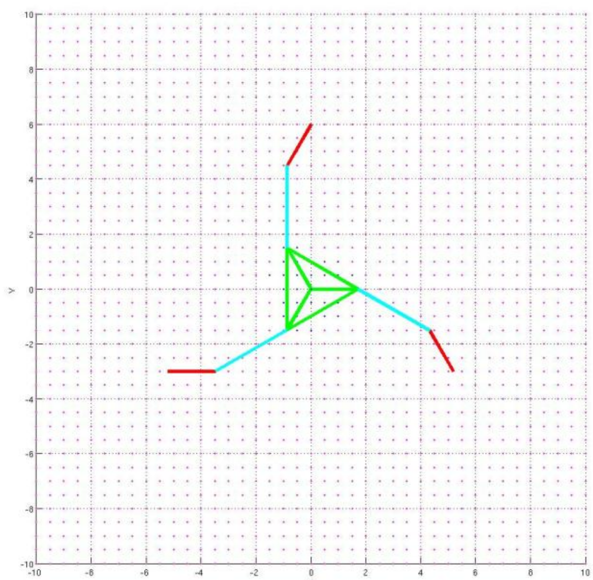


$$(XEE, YEE) = (0.25, 0.433)$$

Please see drop box folder for code.

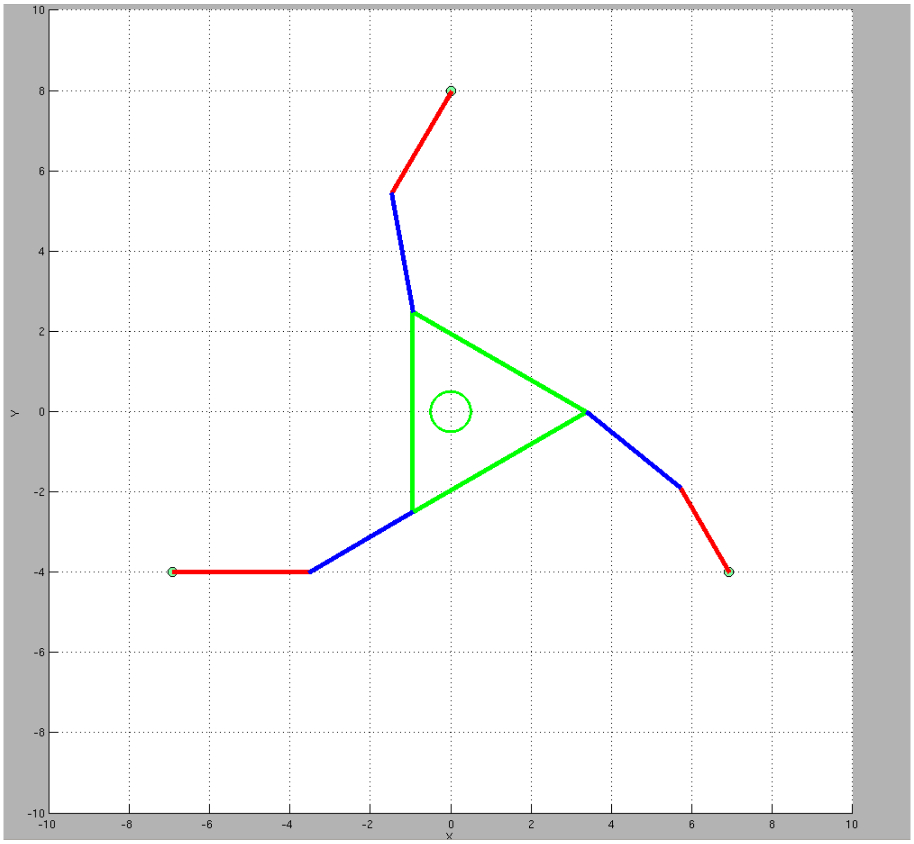
Workspace:





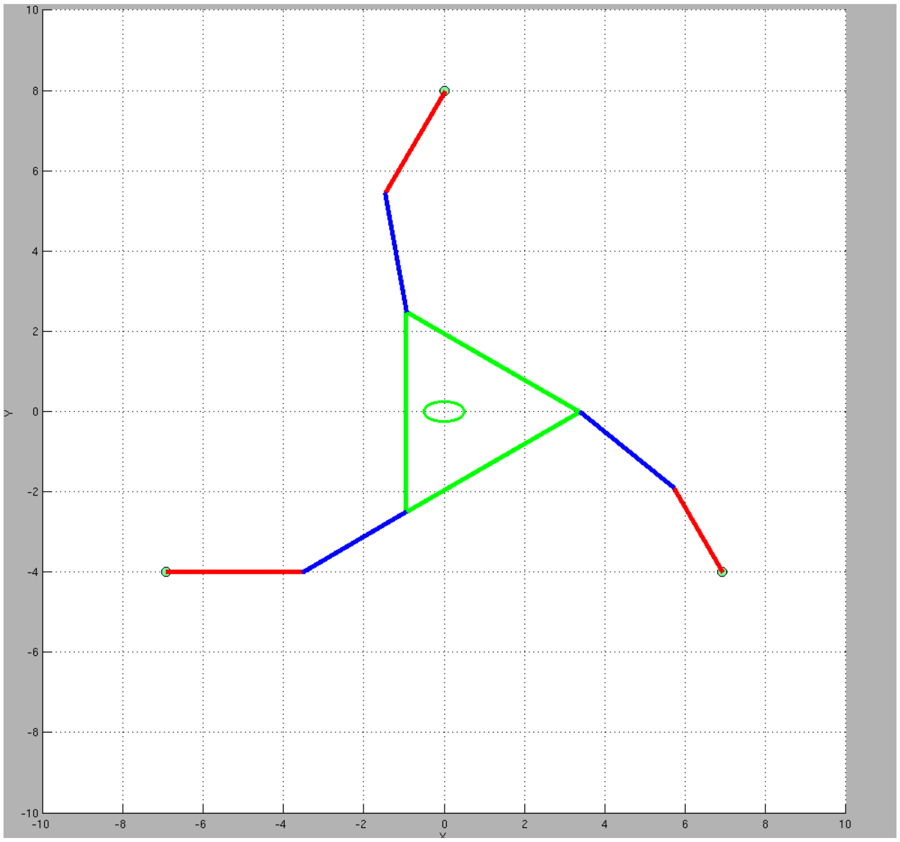
Please see drop box folder for code.

Circle Tracing:



Please see drop box folder for code.

<u>Ellipse Tracing</u>:



Please see drop box folder for code.

Control:

The $Jacobian\ J\ and\ inverse(J)$ are given by

$$J = -inverse(B) * A$$

$$inverse(J) = -inverse(A) * B$$

$$where\ A = \begin{pmatrix} 1 & 0 & -L_{21}\sin\theta_{31} \\ 1 & 0 & -L_{22}\sin\theta_{32} \\ 1 & 0 & -L_{23}\sin\theta_{33} \\ 0 & 1 & L_{21}\cos\theta_{31} \\ 0 & 1 & L_{22}\cos\theta_{32} \\ 0 & 1 & L_{23}\cos\theta_{33} \end{pmatrix}$$

$$and\ B =$$

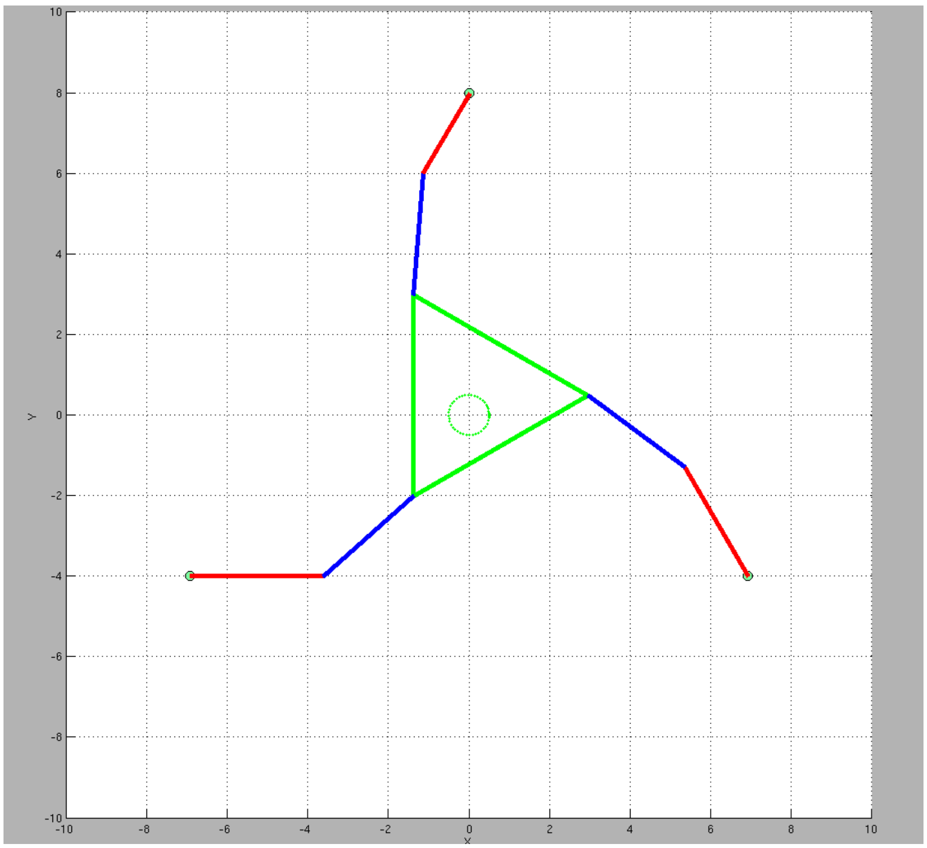
$$\begin{pmatrix} \cos\theta_{11} & 0 & 0 & -L_{11}\sin\theta_{21} & 0 & 0 \\ 0 & \cos\theta_{12} & 0 & 0 & -L_{12}\sin\theta_{22} & 0 \\ 0 & 0 & \cos\theta_{13} & 0 & 0 & -L_{13}\sin\theta_{23} \\ \sin\theta_{11} & 0 & 0 & L_{11}\cos\theta_{21} & 0 & 0 \\ 0 & \sin\theta_{12} & 0 & 0 & L_{12}\cos\theta_{22} & 0 \\ 0 & 0 & \sin\theta_{13} & 0 & 0 & L_{13}\cos\theta_{23} \end{pmatrix}$$

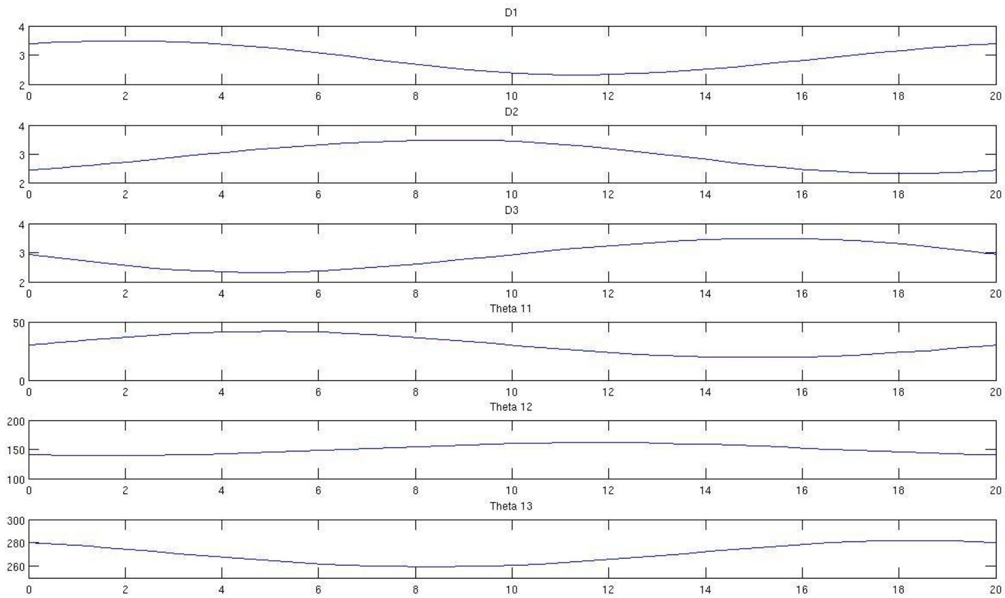
1) Open Loop Control:

Open loop control was achieved using the $matlab\ ode45$ function.

$$\dot{\theta}_{openloop} = inverse(J) * \begin{pmatrix} \dot{X}_{EE} \\ \dot{Y}_{EE} \\ \dot{\phi}_{EE} \end{pmatrix}$$

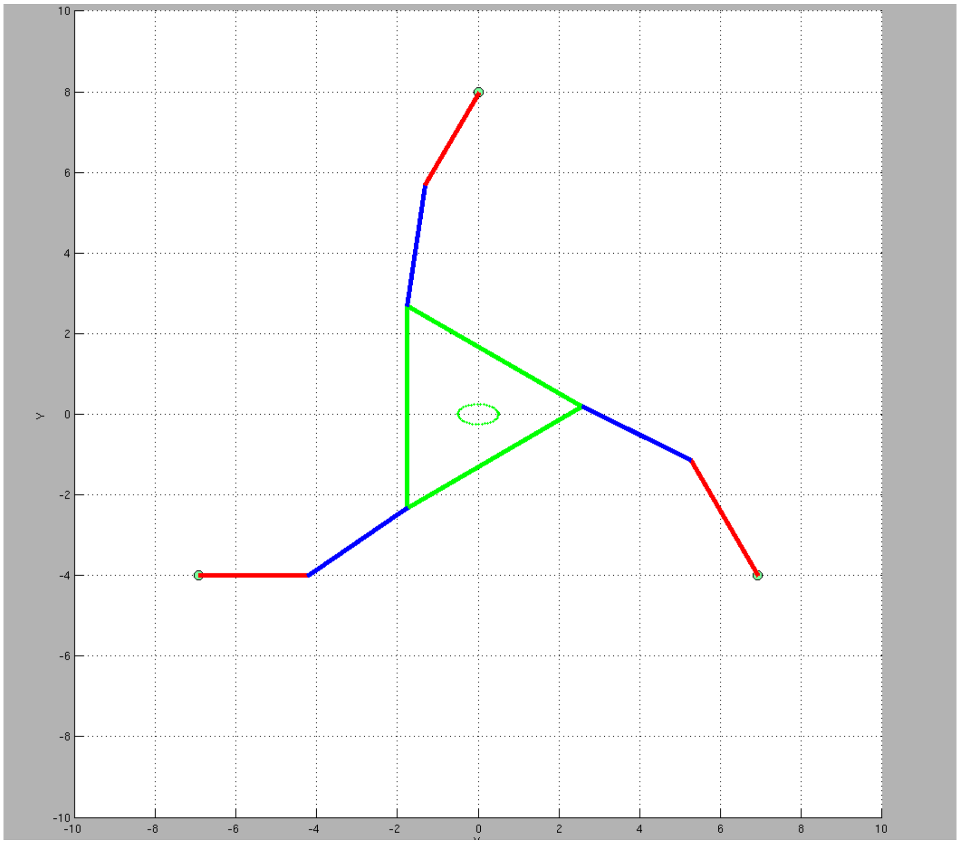
$$\dot{\theta}_{openloop} = -inverse(B) * A * \begin{pmatrix} \dot{X}_{EE} \\ \dot{Y}_{EE} \\ \dot{\emptyset}_{EE} \end{pmatrix}$$

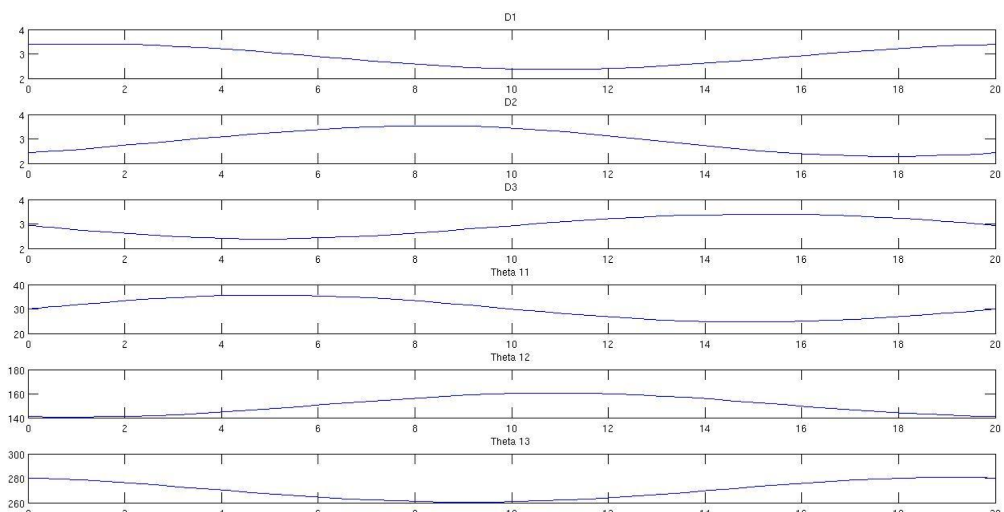




Timeline for $(D_1, D_2, D_3, \theta_{11}, \theta_{12}, \theta_{13})$

Please see drop box folder for code for open loop control.





Timeline for $(D_1, D_2, D_3, \theta_{11}, \theta_{12}, \theta_{13})$

Please see drop box folder for code for open loop control.

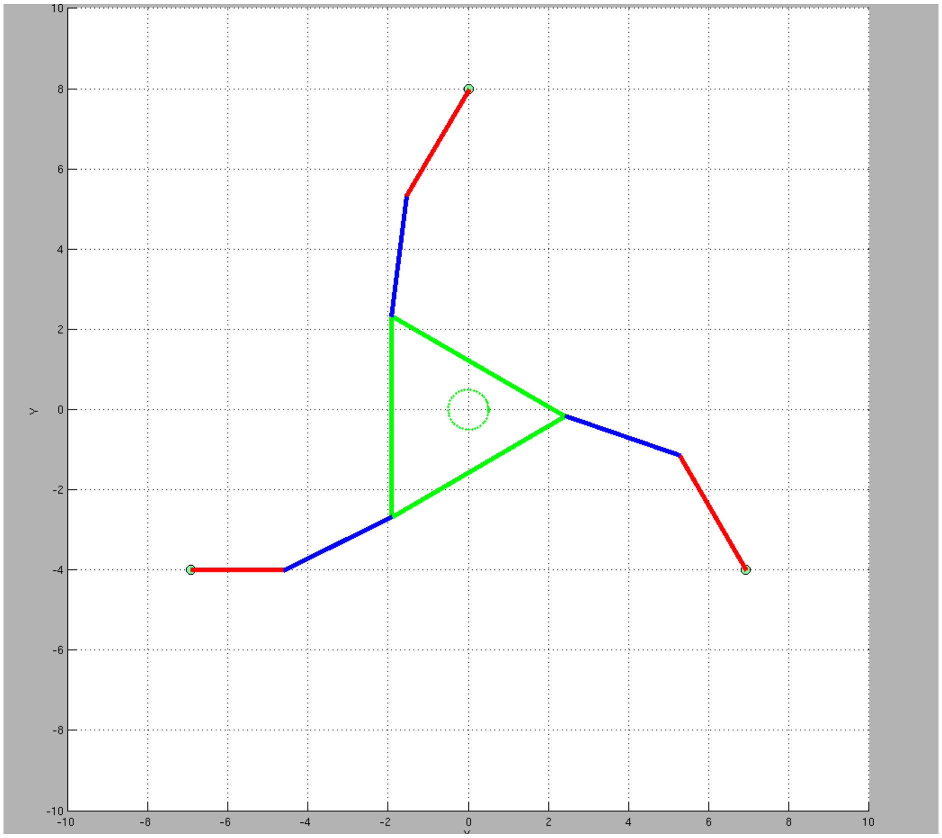
2) Closed Loop Control (Joint Space):

$$\dot{\theta}_{openloop} = -inverse(B) * A * \begin{pmatrix} \dot{X}_{EE} \\ \dot{Y}_{EE} \\ \dot{\emptyset}_{EE} \end{pmatrix}$$

$$\dot{\theta}_{total} = \dot{\theta}_{openloop} + K * \left\{ \begin{pmatrix} D_{1desired} \\ D_{2desired} \\ D_{3desired} \\ \theta_{11desired} \\ \theta_{12desired} \\ \theta_{13desired} \end{pmatrix} - \begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ \theta_{11} \\ \theta_{12} \\ \theta_{13} \end{pmatrix} \right\}$$

$$\dot{\theta}_{total} = -inverse(B) * A * \begin{pmatrix} \dot{X}_{EE} \\ \dot{Y}_{EE} \\ \dot{\emptyset}_{EE} \end{pmatrix} + K$$

$$* \left\{ \begin{pmatrix} D_{1desired} \\ D_{2desired} \\ D_{3desired} \\ \theta_{11desired} \\ \theta_{12desired} \\ \theta_{13desired} \end{pmatrix} - \begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ \theta_{11} \\ \theta_{12} \\ \theta_{13} \end{pmatrix} \right\}$$



10
8
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8
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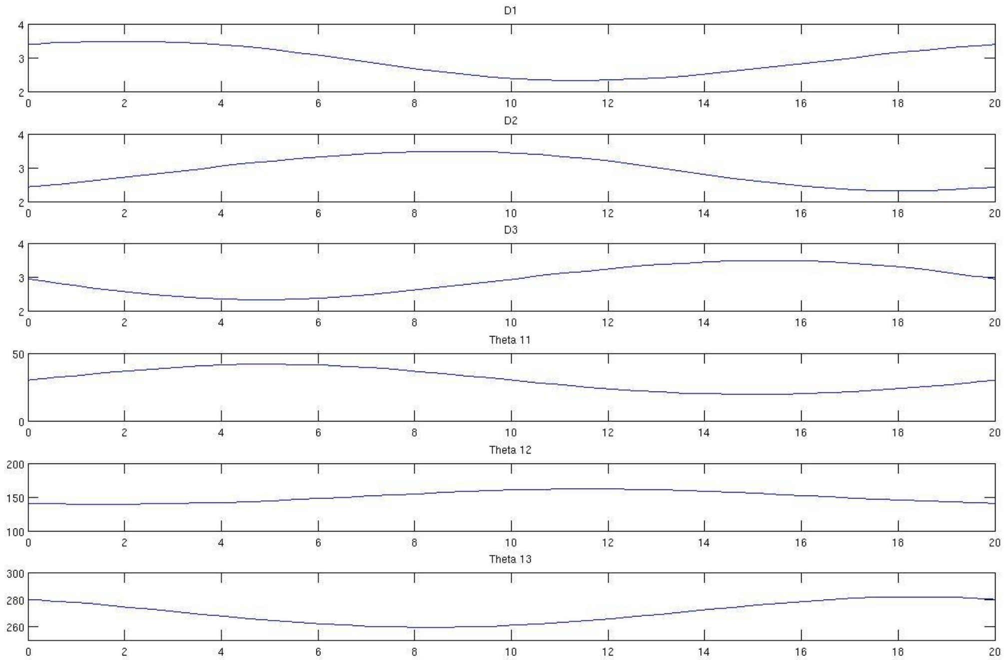
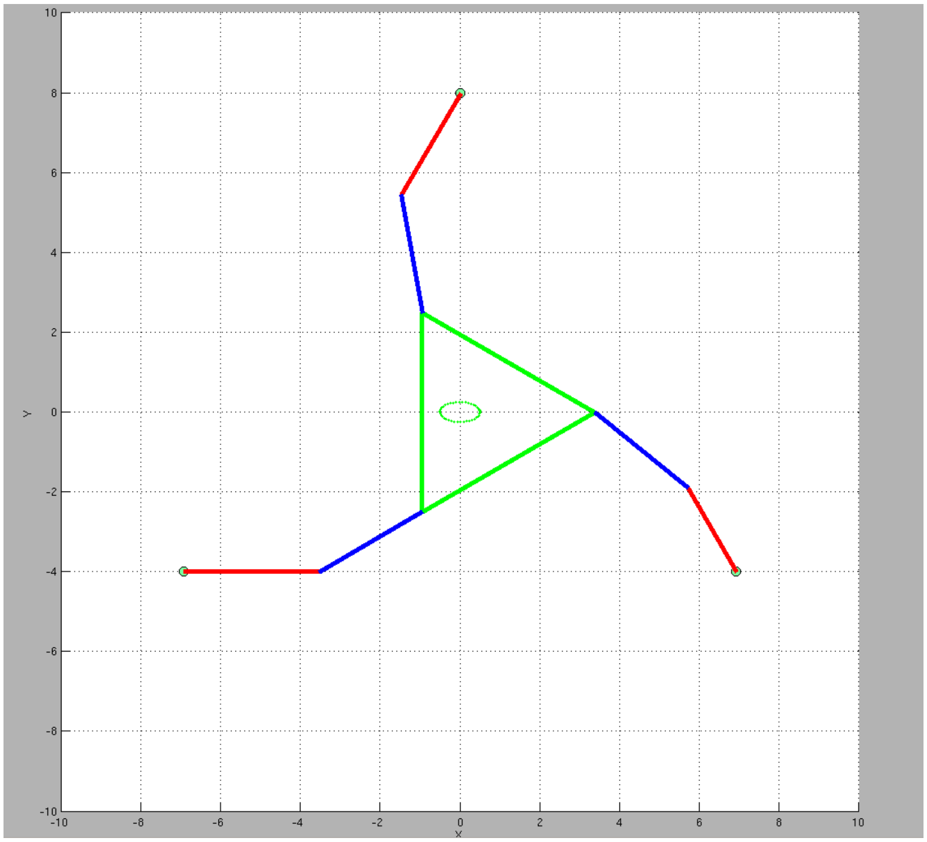
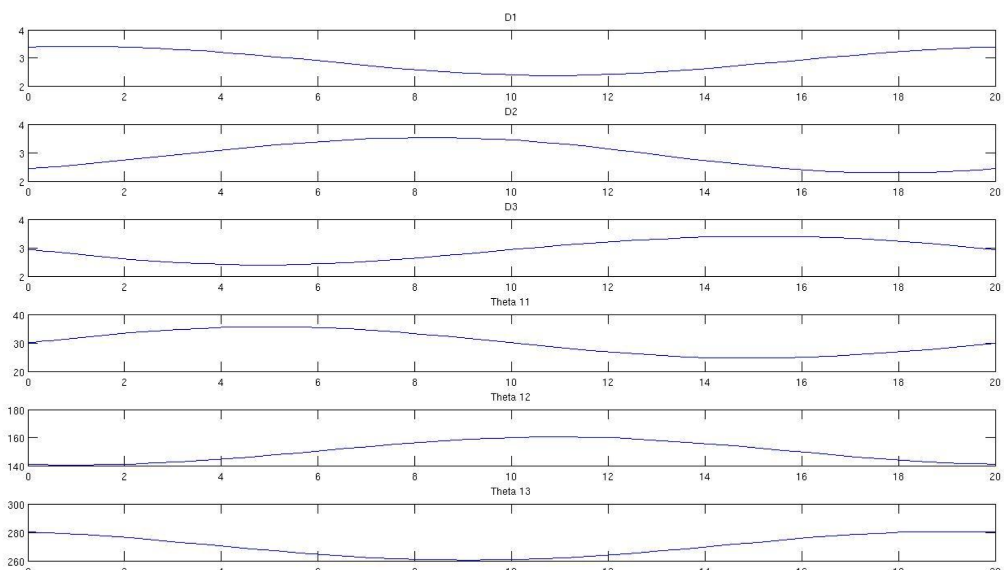


Timeline for $(D_1, D_2, D_3, \theta_{11}, \theta_{12}, \theta_{13})$

Please see drop box folder for code for closed loop control.





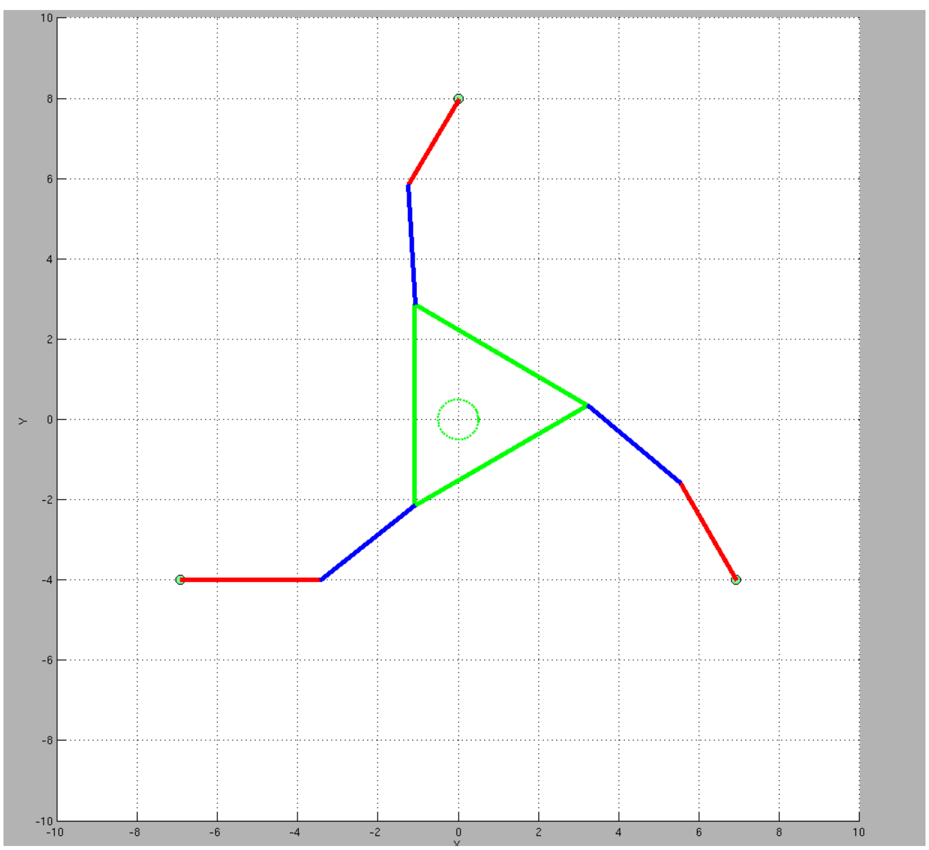
Timeline for $(D_1, D_2, D_3, \theta_{11}, \theta_{12}, \theta_{13})$

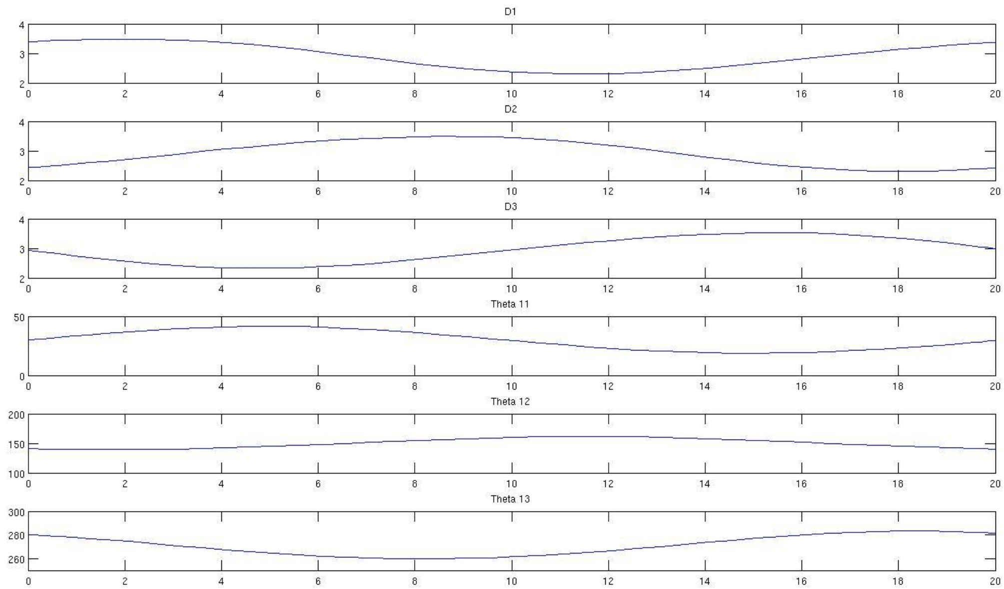
Please see drop box folder for code for closed loop control.

3) Closed Loop Control (Task Space):

$$\dot{\theta}_{closed} = inverse(J) * \left\{ \begin{pmatrix} \dot{X}_{EE} \\ \dot{Y}_{EE} \\ \dot{\emptyset}_{EE} \end{pmatrix} + K * \left\{ \begin{pmatrix} X_{EE\,desired} \\ Y_{EE\,desired} \\ \emptyset_{EE\,desired} \end{pmatrix} - \begin{pmatrix} X_{EE} \\ Y_{EE} \\ \emptyset_{EE} \end{pmatrix} \right\} \right\}$$

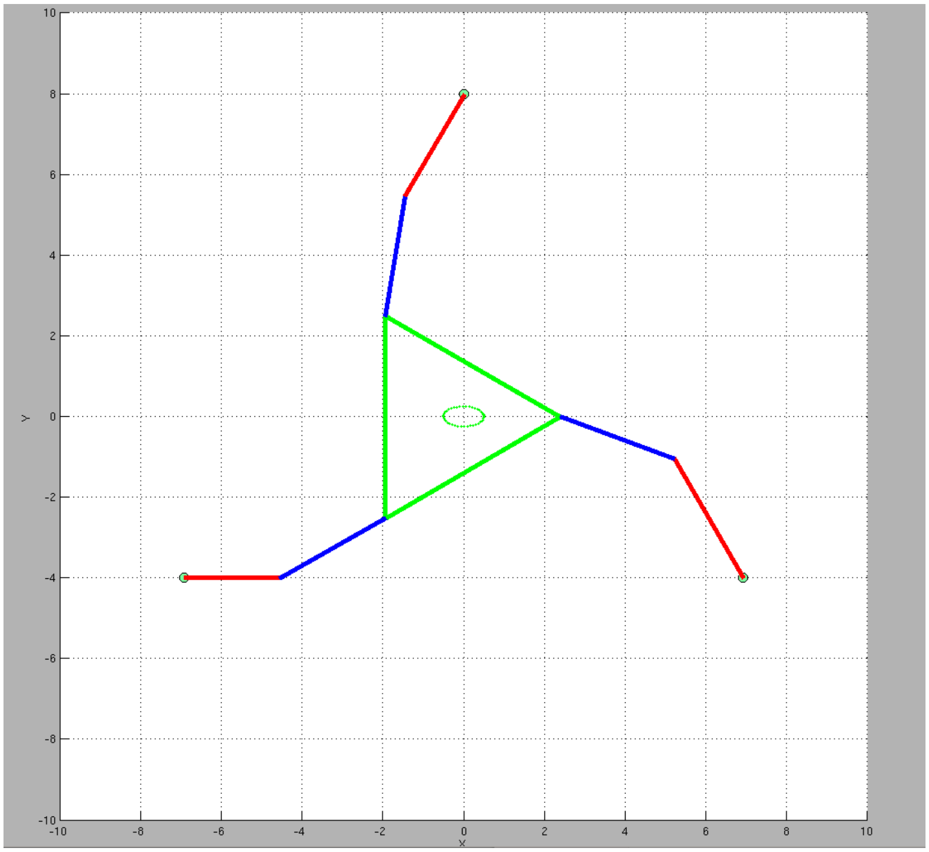
$$\dot{\theta}_{closed} = -inverse(B) * A * \left\{ \begin{pmatrix} \dot{X}_{EE} \\ \dot{Y}_{EE} \\ \dot{\emptyset}_{EE} \end{pmatrix} + K * \left\{ \begin{pmatrix} X_{EE\,desired} \\ Y_{EE\,desired} \\ \emptyset_{EE\,desired} \end{pmatrix} - \begin{pmatrix} X_{EE} \\ Y_{EE} \\ \emptyset_{EE} \end{pmatrix} \right\} \right\}$$

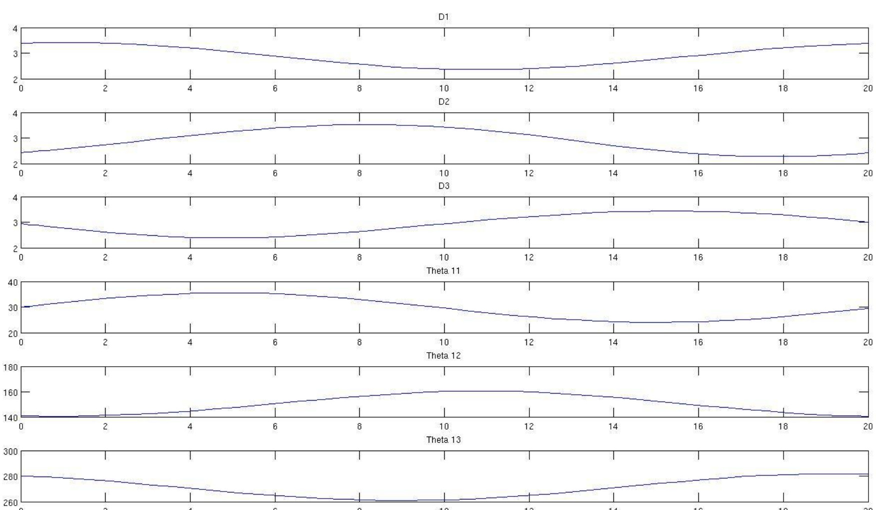




Timeline for $(D_1, D_2, D_3, \theta_{11}, \theta_{12}, \theta_{13})$

Please see drop box folder for code for closed loop control.





Timeline for $(D_1, D_2, D_3, \theta_{11}, \theta_{12}, \theta_{13})$

Please see drop box folder for code for closed loop control.

Manipulability:

The $Jacobian\ J\ and\ inverse(J)$ are given by

$$J = -inverse(B) * A$$

$$inverse(J) = -inverse(A) * B$$

$$where\ A = \begin{pmatrix} 1 & 0 & -L_{21}\sin\theta_{31} \\ 1 & 0 & -L_{22}\sin\theta_{32} \\ 1 & 0 & -L_{23}\sin\theta_{33} \\ 0 & 1 & L_{21}\cos\theta_{31} \\ 0 & 1 & L_{22}\cos\theta_{32} \\ 0 & 1 & L_{23}\cos\theta_{33} \end{pmatrix}$$

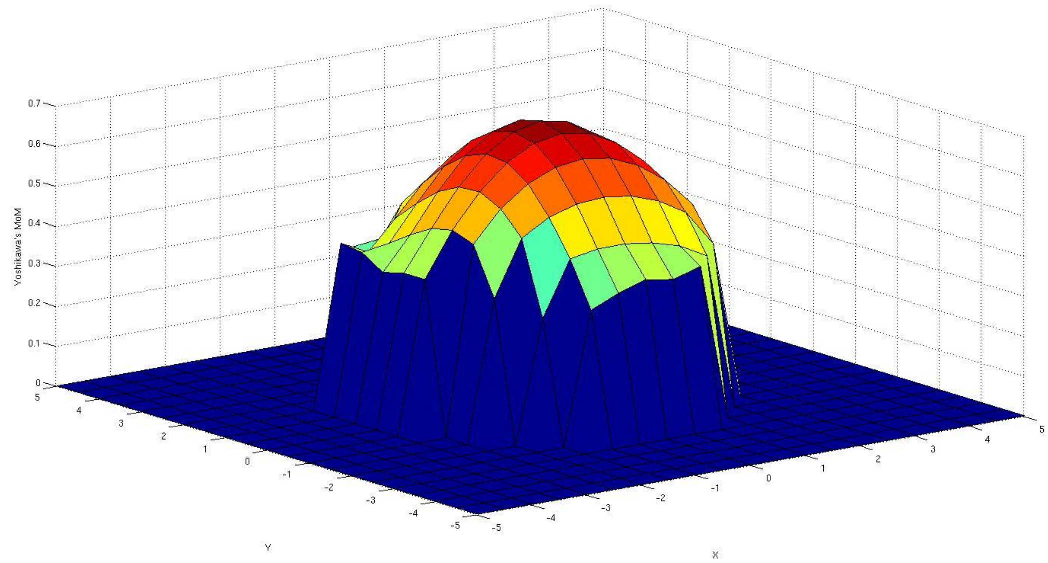
$$and\ B$$

$$= \begin{pmatrix} \cos\theta_{11} & 0 & 0 & -L_{11}\sin\theta_{21} & 0 & 0 \\ 0 & \cos\theta_{12} & 0 & 0 & -L_{12}\sin\theta_{22} & 0 \\ 0 & 0 & \cos\theta_{13} & 0 & 0 & -L_{13}\sin \\ \sin\theta_{11} & 0 & 0 & L_{11}\cos\theta_{21} & 0 & 0 \\ 0 & \sin\theta_{12} & 0 & 0 & L_{12}\cos\theta_{22} & 0 \\ 0 & 0 & \sin\theta_{13} & 0 & 0 & L_{13}\cos\theta \end{pmatrix}$$

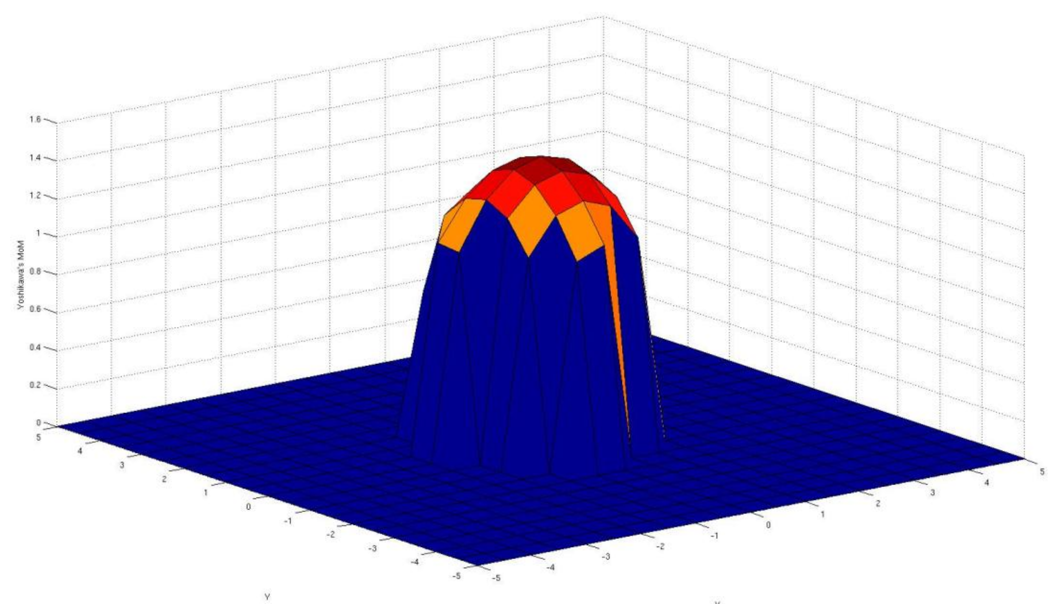
1) Yoshikawa Measure of Manipulability

$$YMOM = \sqrt[2]{\det(J * transpose(J))}$$

For L = 5, R = 8



For L = 3, R = 6



Please see drop box folder for code for Yoshikawa measure of manipulability.

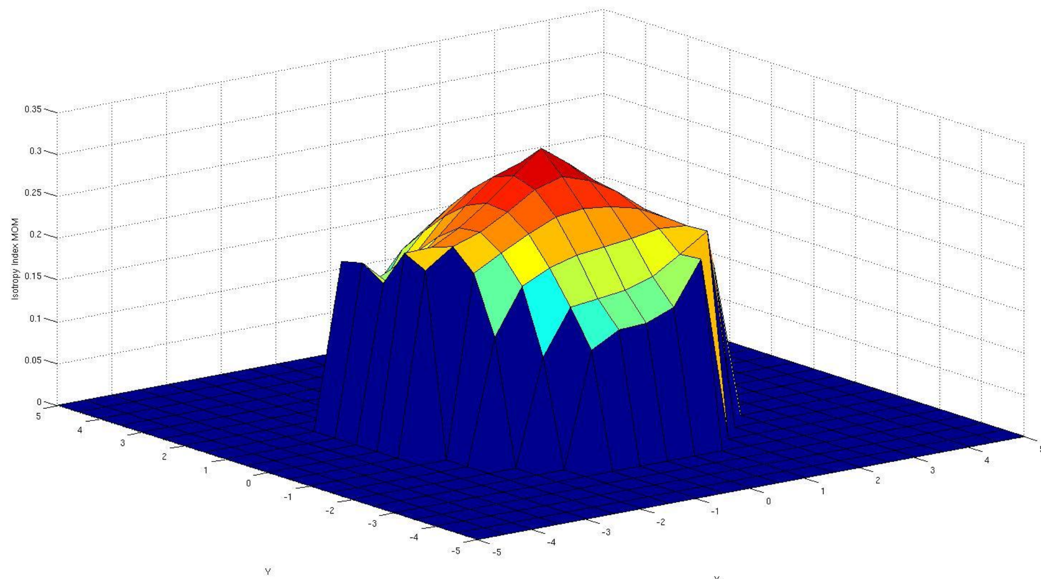
2) Isotropy Index Measure of Manipulability

$$U * Sigma * transpose(V) = SVD(J)$$

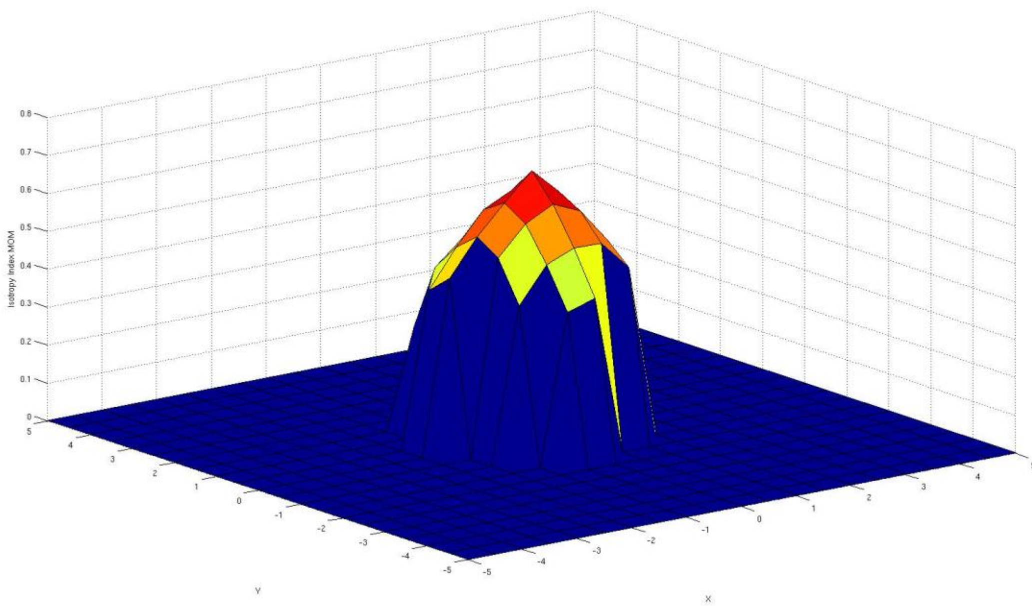
$$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots \geq \sigma_p$$

$$IIMOM = \sigma_p / \sigma_1$$

For L = 5, R = 8



For L = 3, R = 6



Please see drop box folder for code for Isotropy Index measure of manipulability.

Appendix

Forward Kinematics http://www.youtube.com/watch?v=a7xhaL0nNgQ

Inverse Kinematics http://www.youtube.com/watch?v=3Iui26pu7_Q

Workspace http://www.youtube.com/watch?v=X1XuG5JBSes

Circle/Ellipse tracing http://www.youtube.com/watch?v=YgYIR9T2lls

Open Loop Control http://www.youtube.com/watch?v=nNBnEevXJQs

Task Space Closed Loop Control http://www.youtube.com/watch?v=BIOhLbwaimY

Joint Space Closed Loop Control http://www.youtube.com/watch?v=u6Es0GRB6dI

Isotropy Index Measure of Manipulability http://www.youtube.com/watch?v=j6a_oTXjlT0

Yoshikawa Measure of Manipulability http://www.youtube.com/watch?v=DrsB7A8xwBA