PRRR serial chain manipulator

Project Report

MAE 593

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Introduction:

The PRRR serial chain manipulator is redundant. $matlab\ fmincon$ was used to determine the inverse kinematics of the system for a given end effector position.

$$XEE = D + L1\cos\theta_1 + L2\cos\theta_2 + L3\cos\theta_3$$

$$YEE = L1\sin\theta_1 + L2\sin\theta_2 + L3\sin\theta_3$$

We define $f_1(D, \theta_1, \theta_2, \theta_3)$ and $f_2(D, \theta_1, \theta_2, \theta_3)$ as below and try to minimize the objective function $g(D, \theta_1, \theta_2, \theta_3)$ as below:-

$$f_1(D, \theta_1, \theta_2, \theta_3) = D + L1\cos\theta_1 + L2\cos\theta_2 + L3\cos\theta_3 - XEE$$

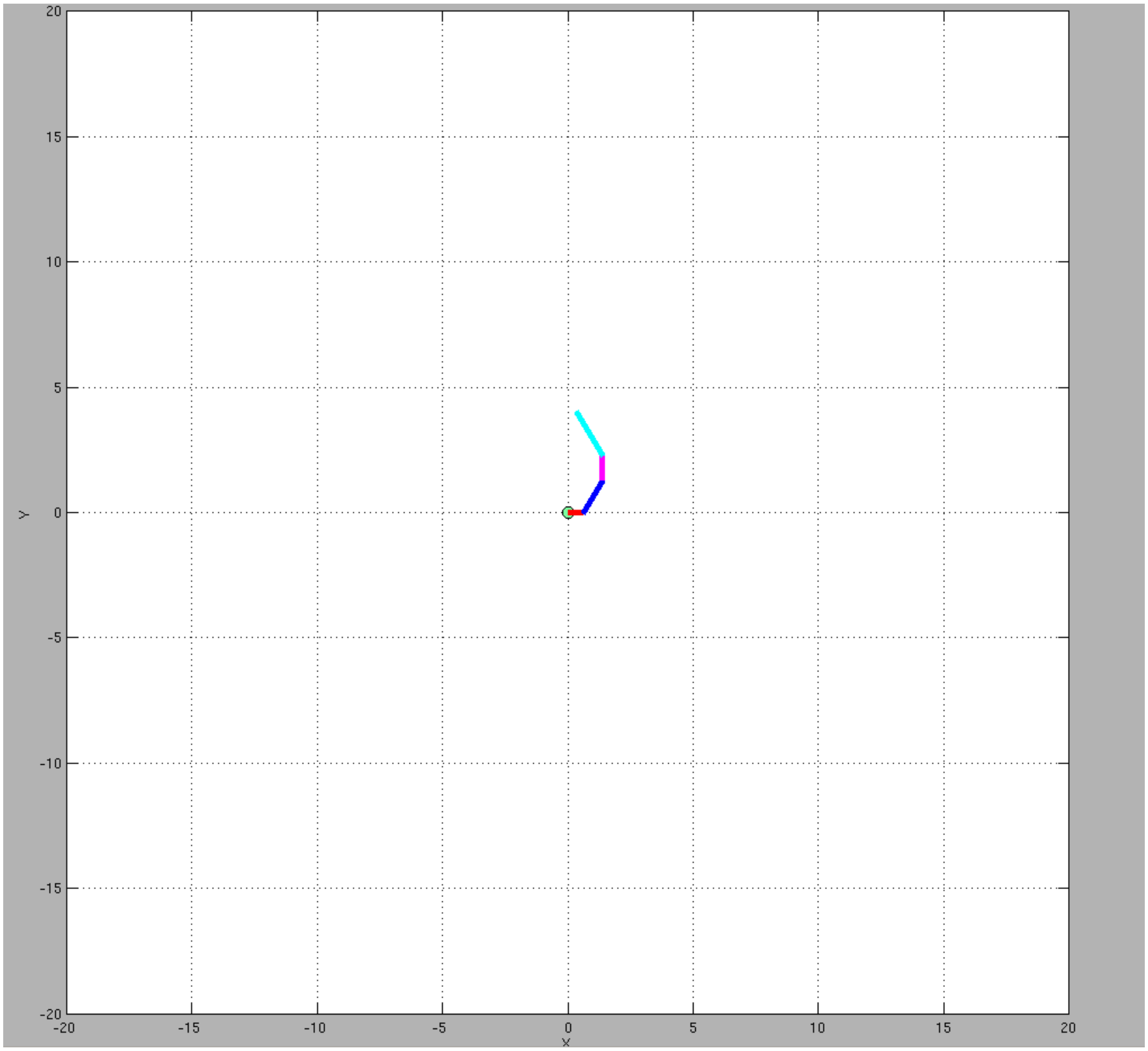
$$f_2(D, \theta_1, \theta_2, \theta_3) = L1\sin\theta_1 + L2\sin\theta_2 + L3\sin\theta_3 - YEE$$

$$g(D, \theta_1, \theta_2, \theta_3) = \sqrt[2]{(f_1(D, \theta_1, \theta_2, \theta_3)^2 + f_2(D, \theta_1, \theta_2, \theta_3)^2)}$$

We use $\mathrm{fmincon}$ wherein we try to

$$\min(g(D, \theta_1, \theta_2, \theta_3)) \text{ such that } 0 \le D \le D_{max}$$

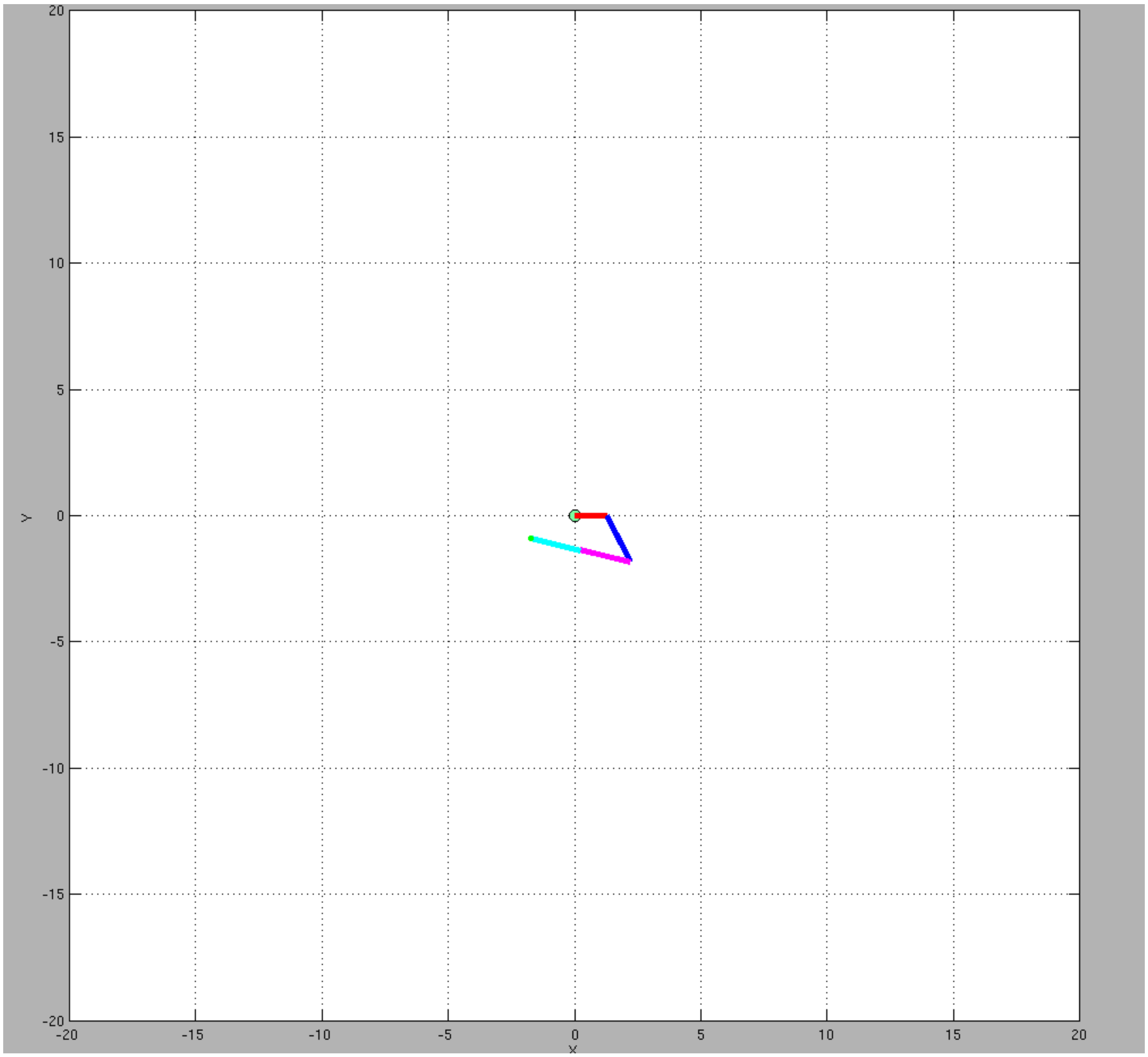
<u>Forward Kinematics</u>:



$$(D, L1, L2, L3) = (0.6, 1.5, 1, 2), (\theta_1, \theta_2, \theta_3) = (60, 90, 120)$$

Please see drop box folder for code.

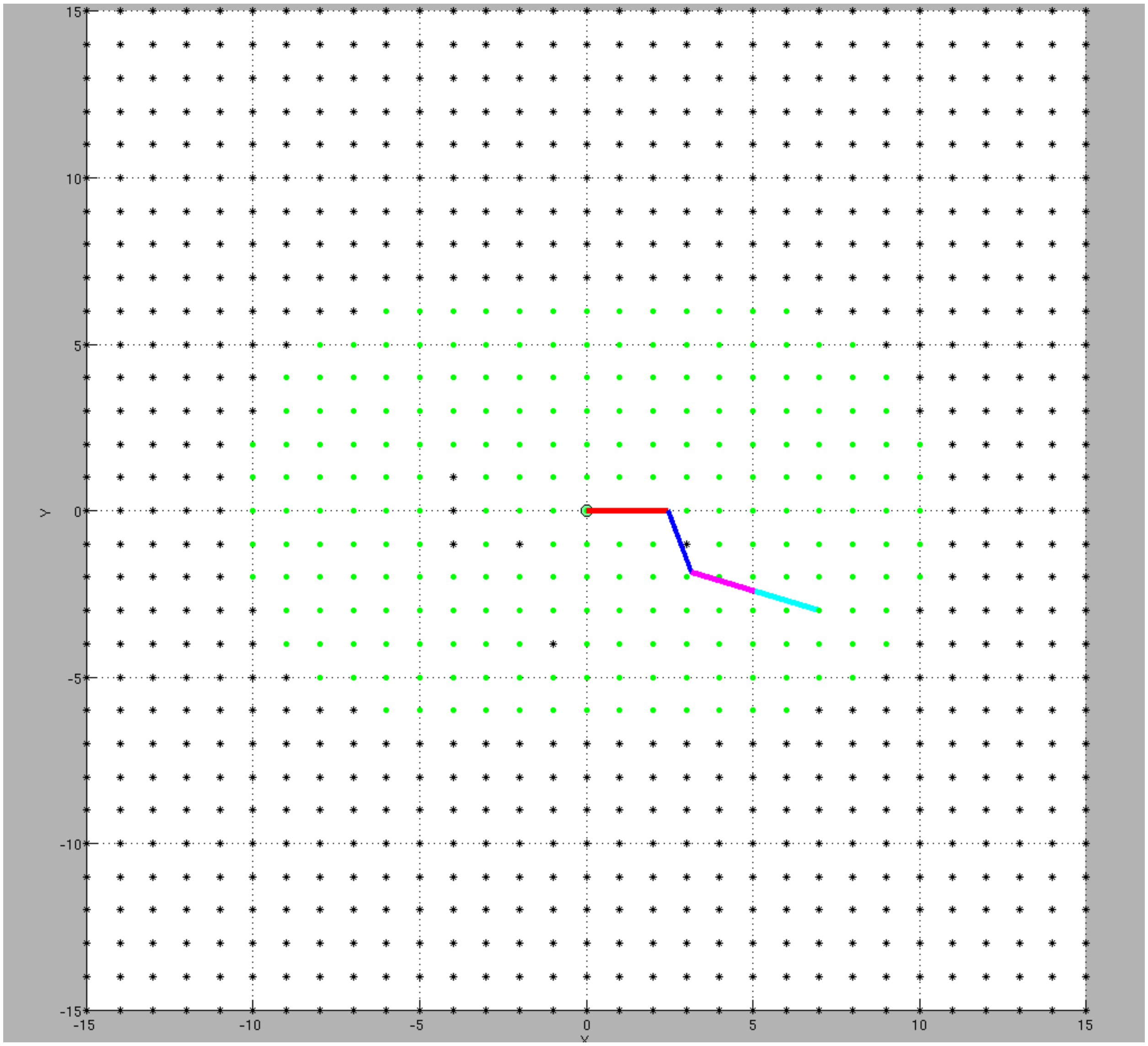
Inverse Kinematics:



$$(XEE, YEE) = (-2, -1)$$

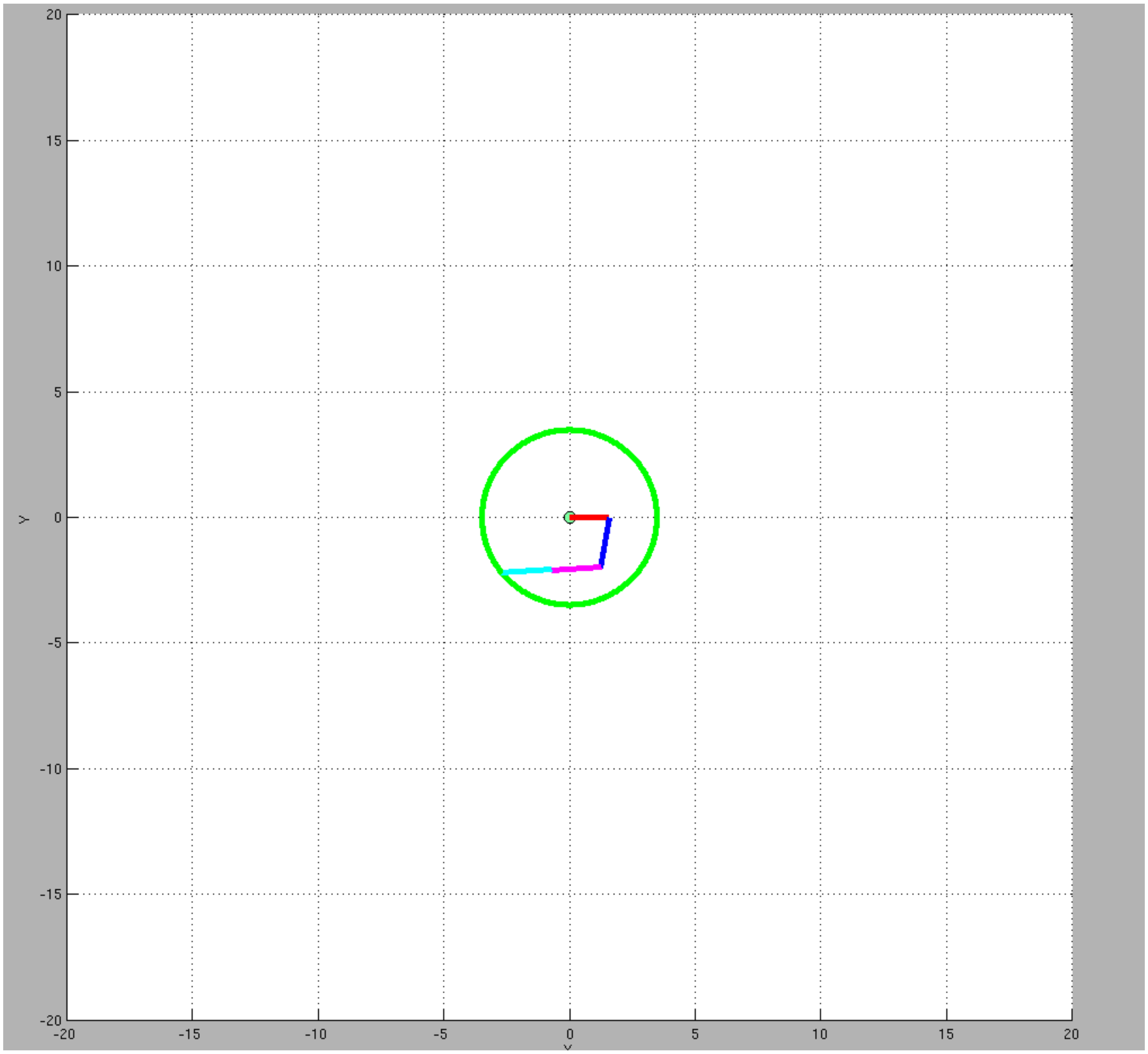
Please see drop box folder for code.

<u>Workspace</u>:



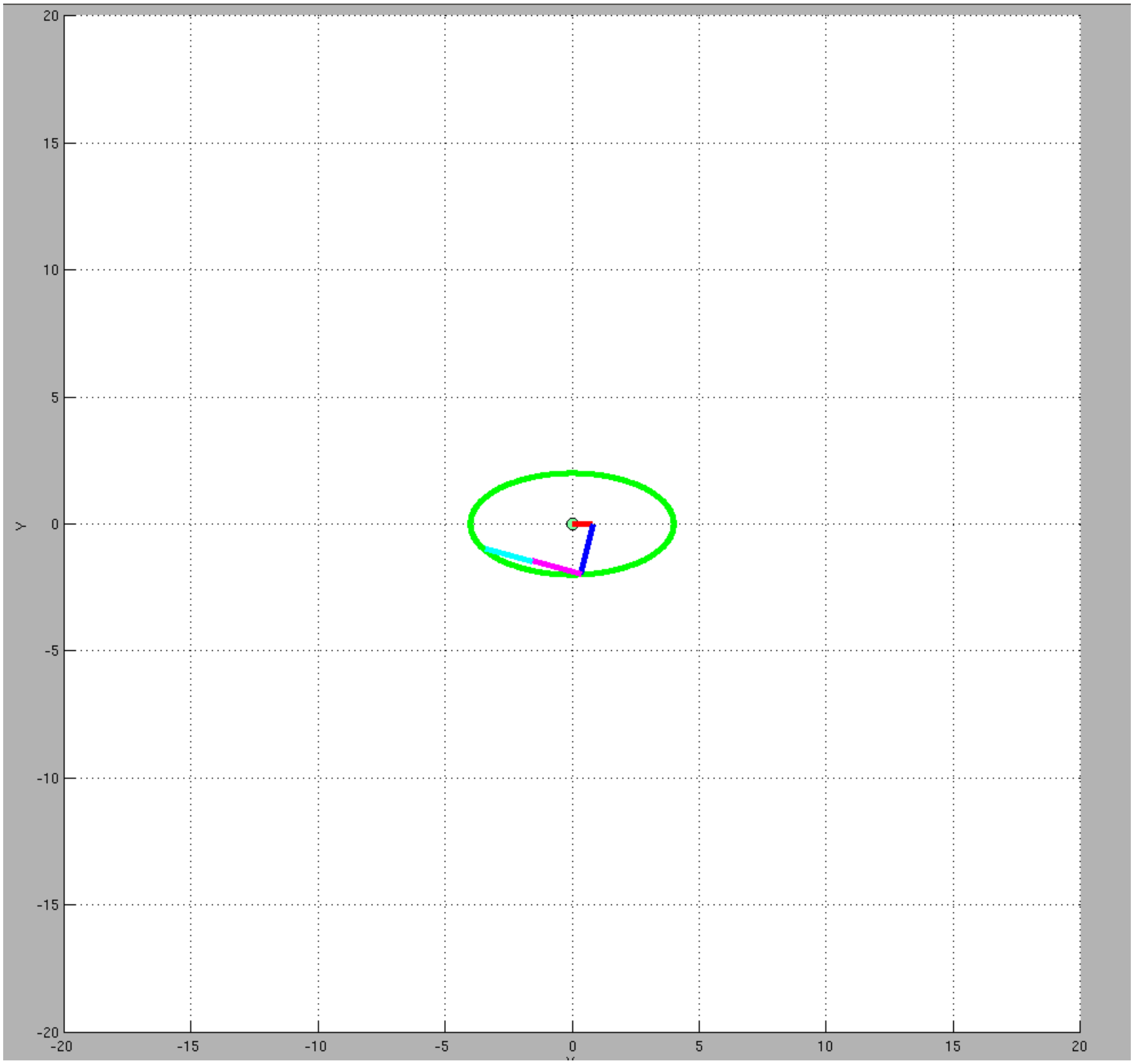
Please see drop box folder for code.

<u>Circle Tracing</u>:



Please see drop box folder for code.

<u>Ellipse Tracing</u>:



Please see drop box folder for code.

Control:

The $Jacobian\ J$ is given by

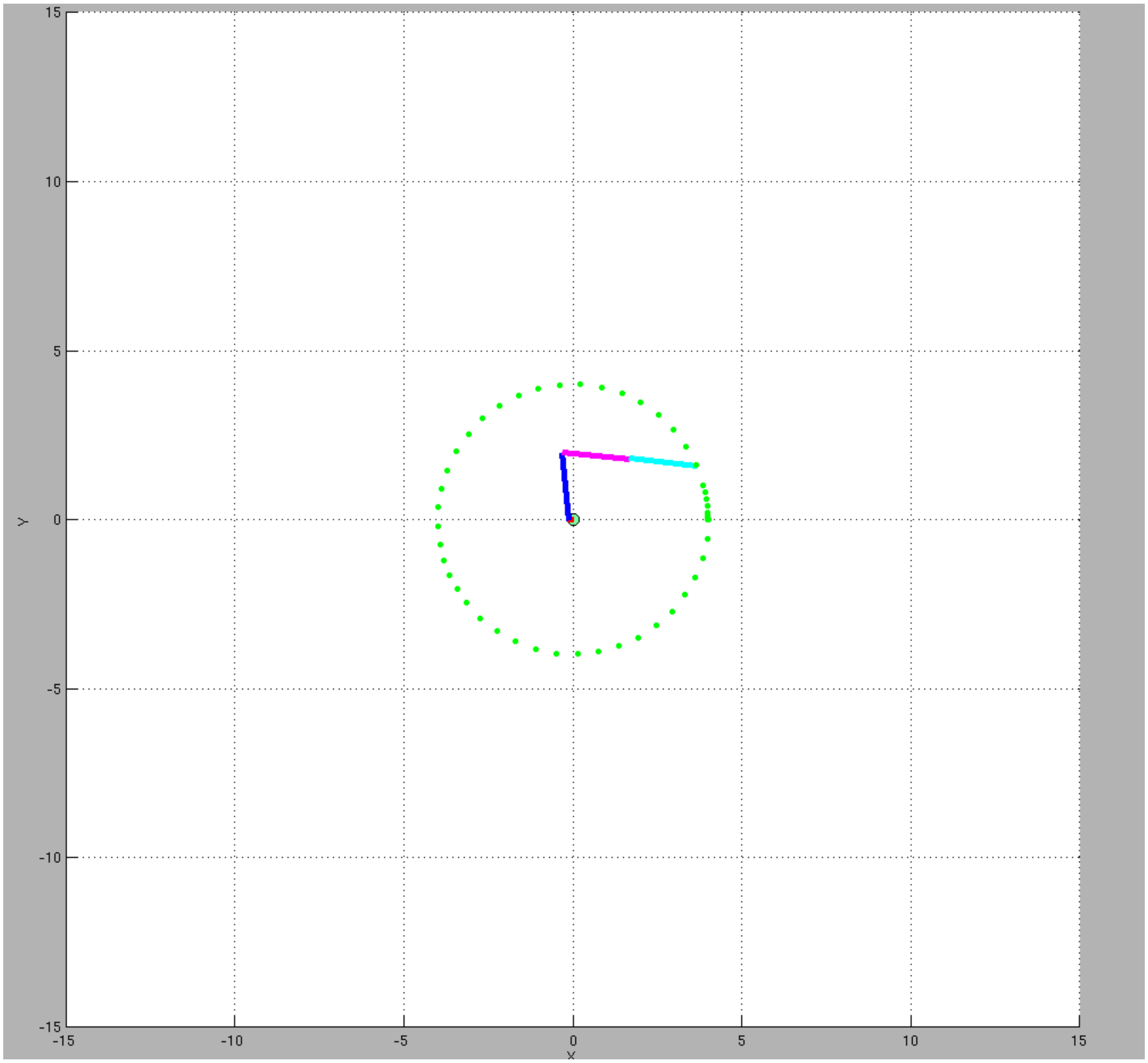
$$J = \begin{pmatrix} {df_1}/{dD} & {df_1}/{d\theta_1} & {df_1}/{d\theta_2} & {df_1}/{d\theta_3} \\ {df_2}/{dD} & {df_2}/{d\theta_1} & {df_2}/{d\theta_2} & {df_2}/{d\theta_3} \end{pmatrix}$$

1) Open Loop Control:

Open loop control was achieved using the $matlab\ ode45$ function.

$$pinvJ = transpose(J) * inverse(J * transpose(J))$$

$$\dot{\theta}_{openloop} = pinvJ * \begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix}$$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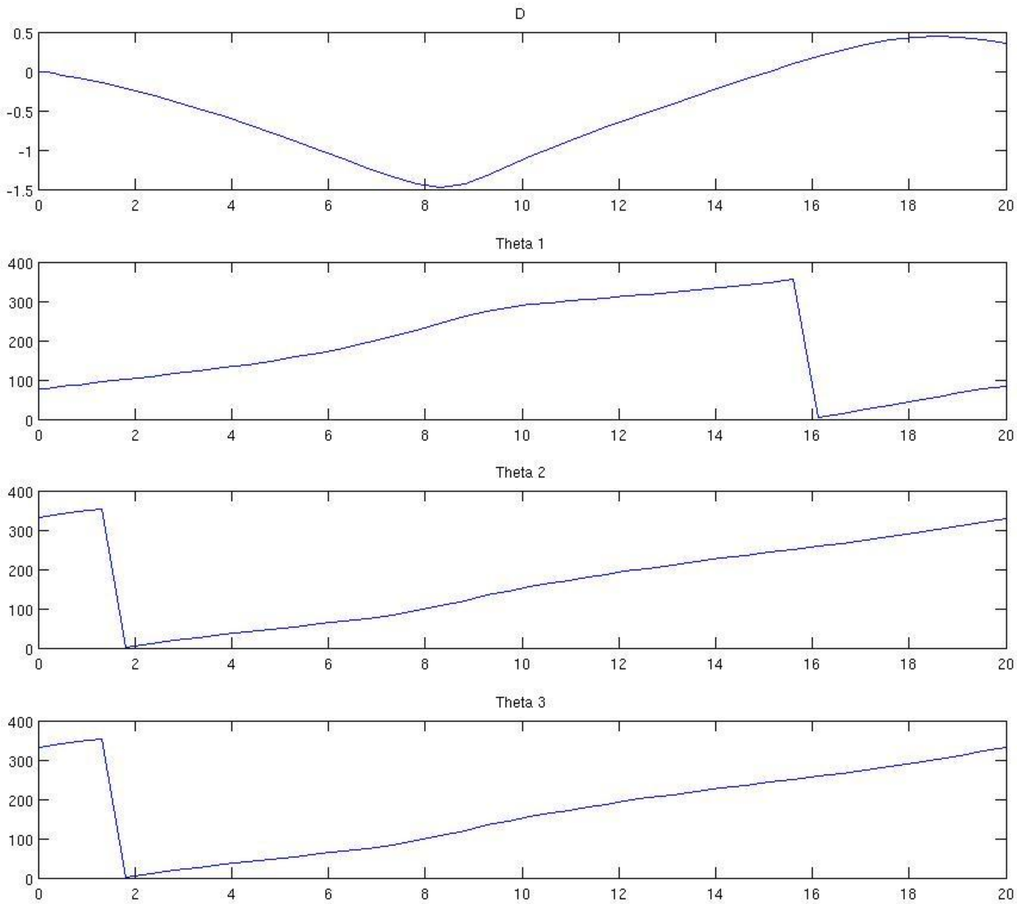
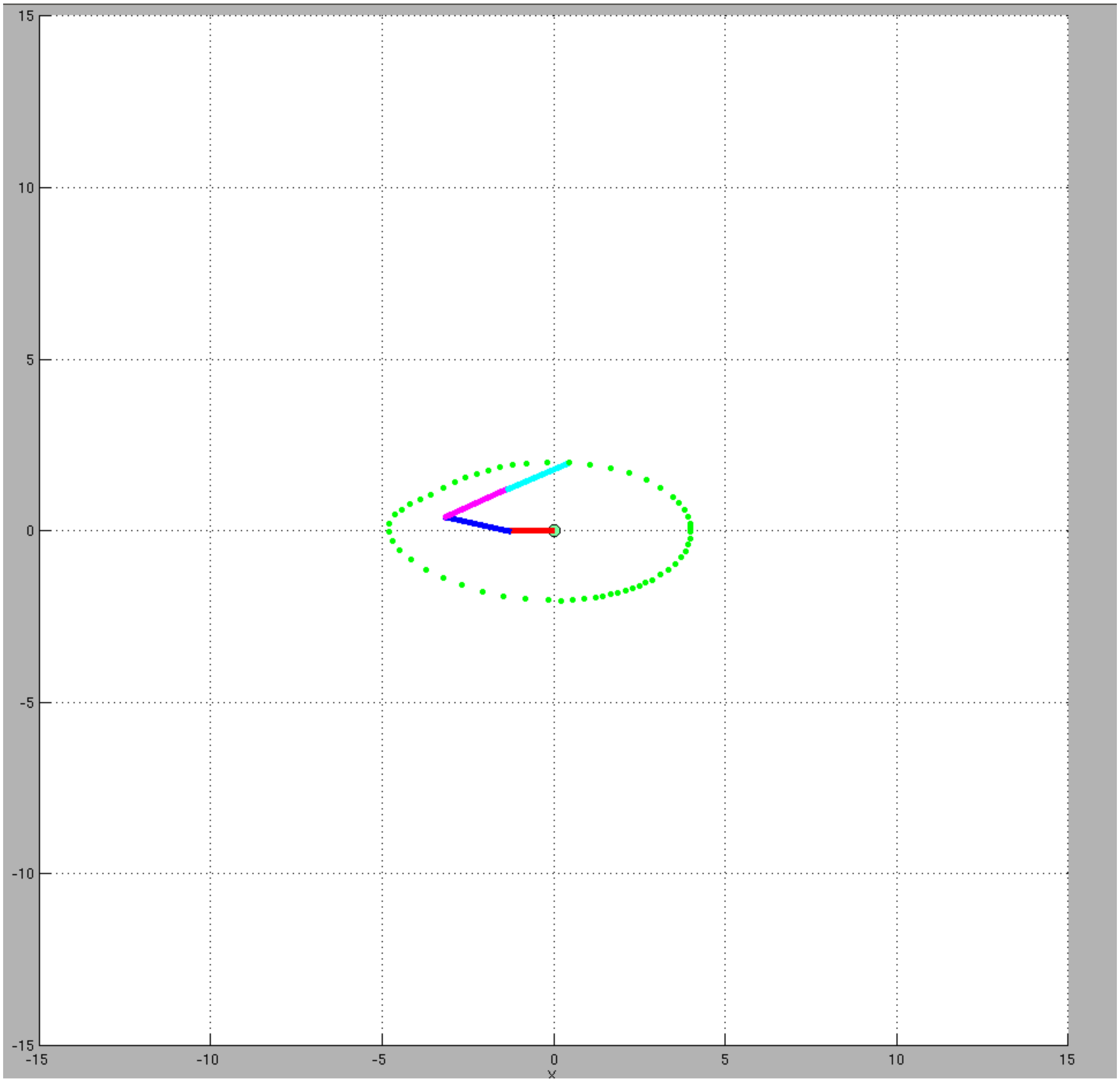


Timeline for $(D, \theta_1, \theta_2, \theta_3)$

Please see drop box folder for code for open loop control.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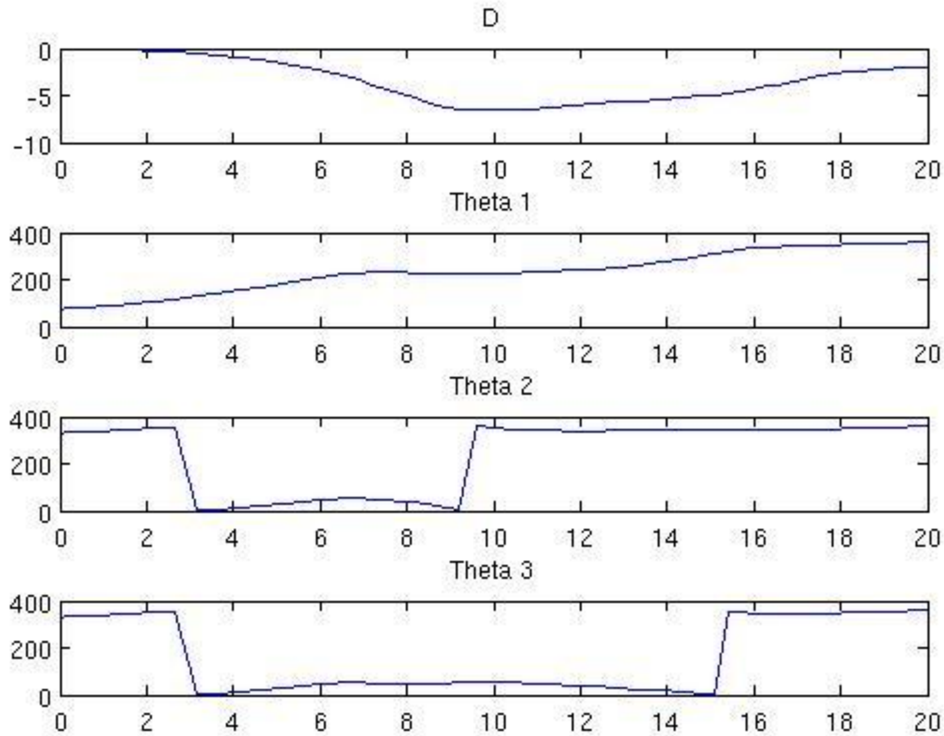


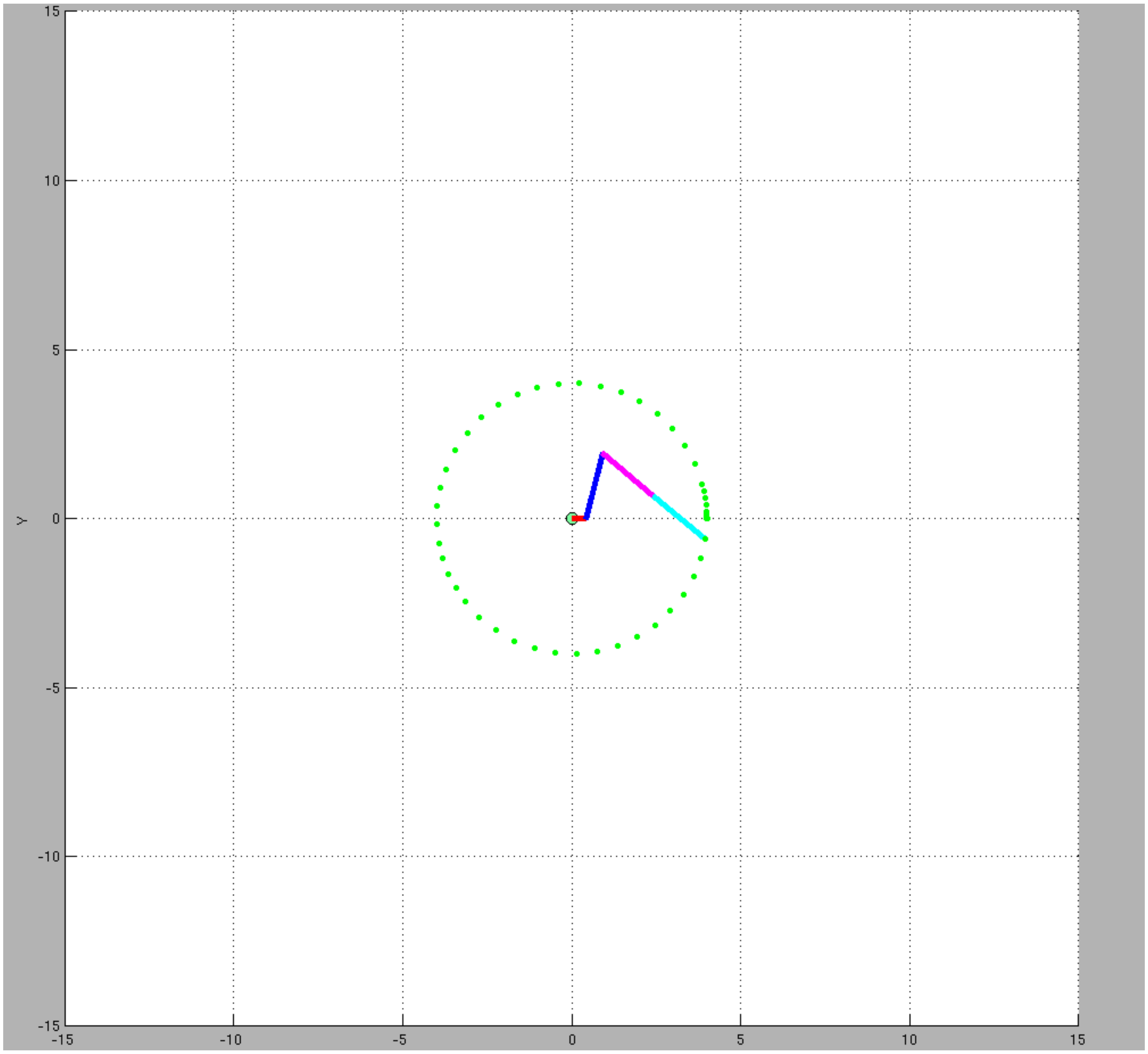
Timeline for $(D, \theta_1, \theta_2, \theta_3)$

Please see drop box folder for code for open loop control.

2) Closed Loop Control (Joint Space):

$$\dot{\theta}_{total} = \dot{\theta}_{openloop} + K * \left\{ \begin{pmatrix} D_{desired} \\ \theta_{1desired} \\ \theta_{2desired} \\ \theta_{3desired} \end{pmatrix} - \begin{pmatrix} D \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} \right\}$$

$$\dot{\theta}_{total} = pinvJ * \begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} + K * \left\{ \begin{pmatrix} D_{desired} \\ \theta_{1desired} \\ \theta_{2desired} \\ \theta_{3desired} \end{pmatrix} - \begin{pmatrix} D \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} \right\}$$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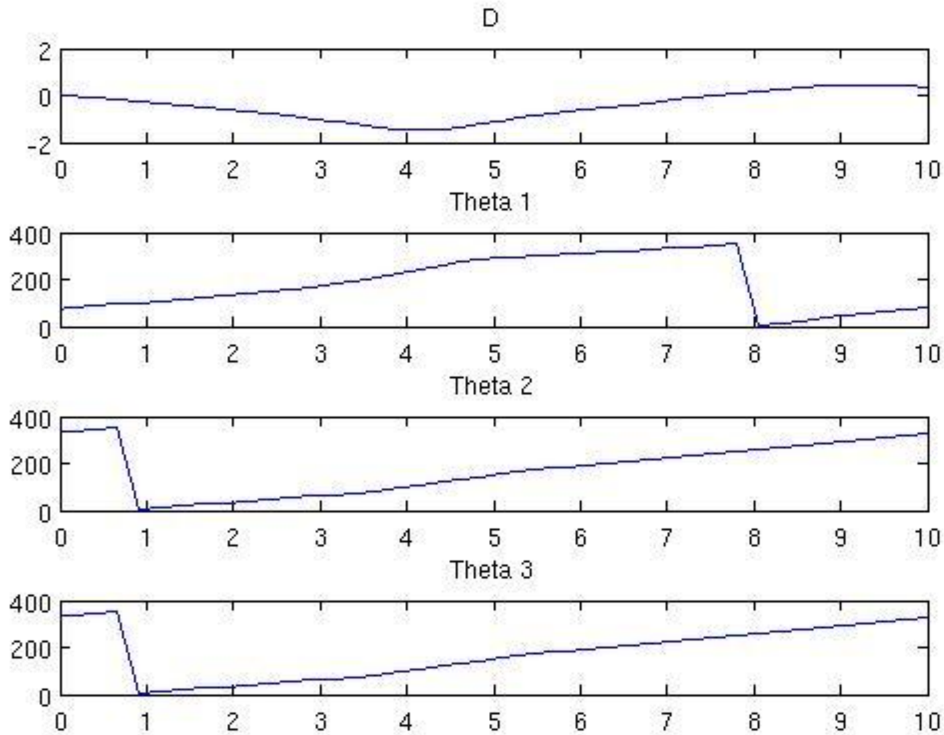
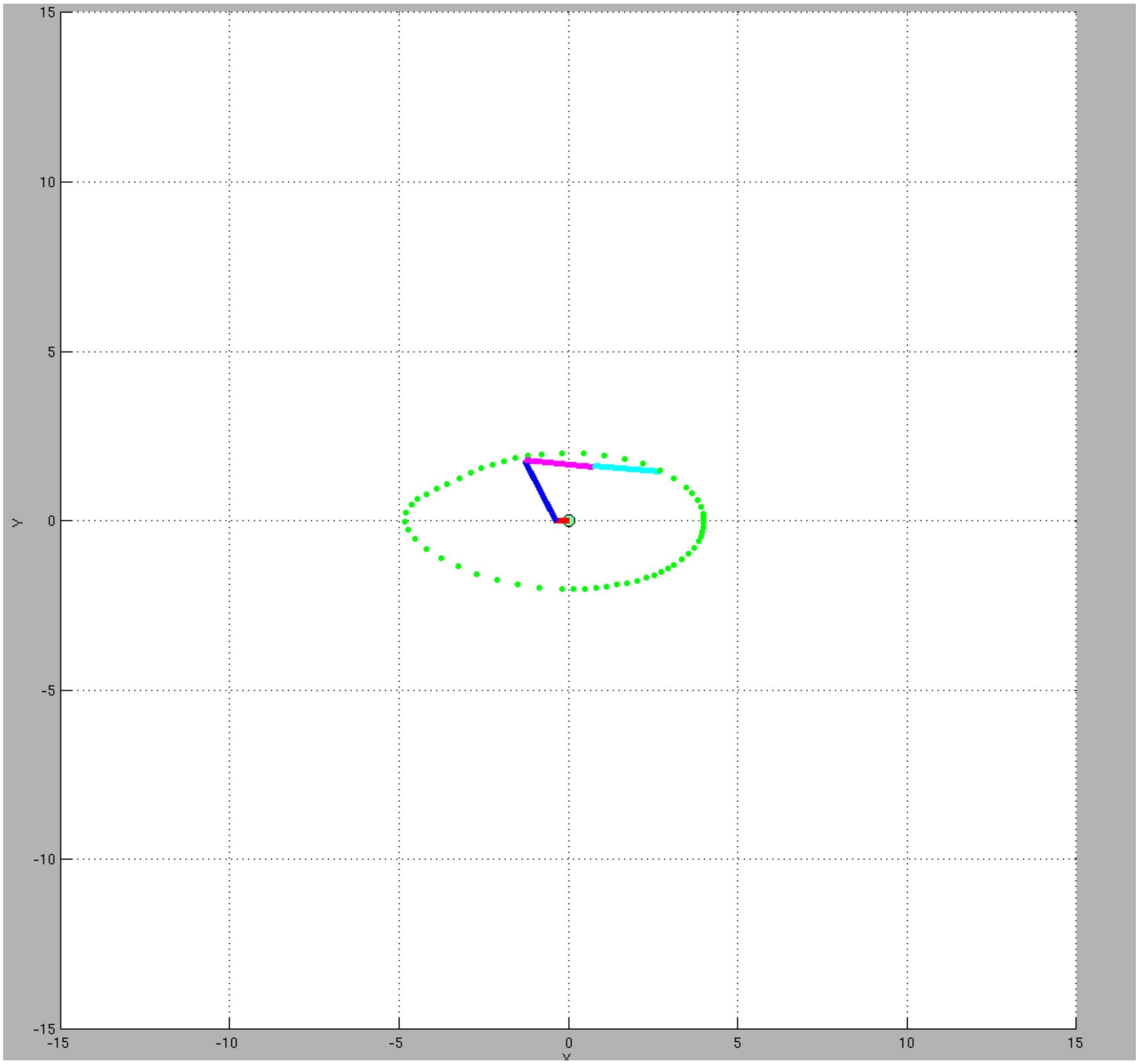


Timeline for $(D, \theta_1, \theta_2, \theta_3)$

Please see drop box folder for code for closed loop control.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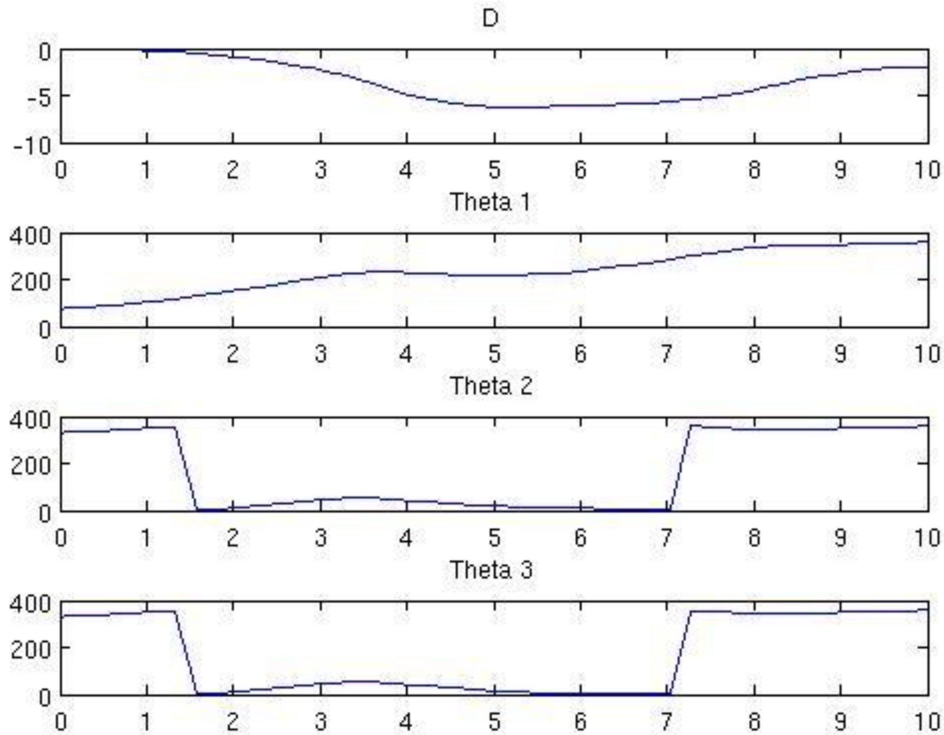
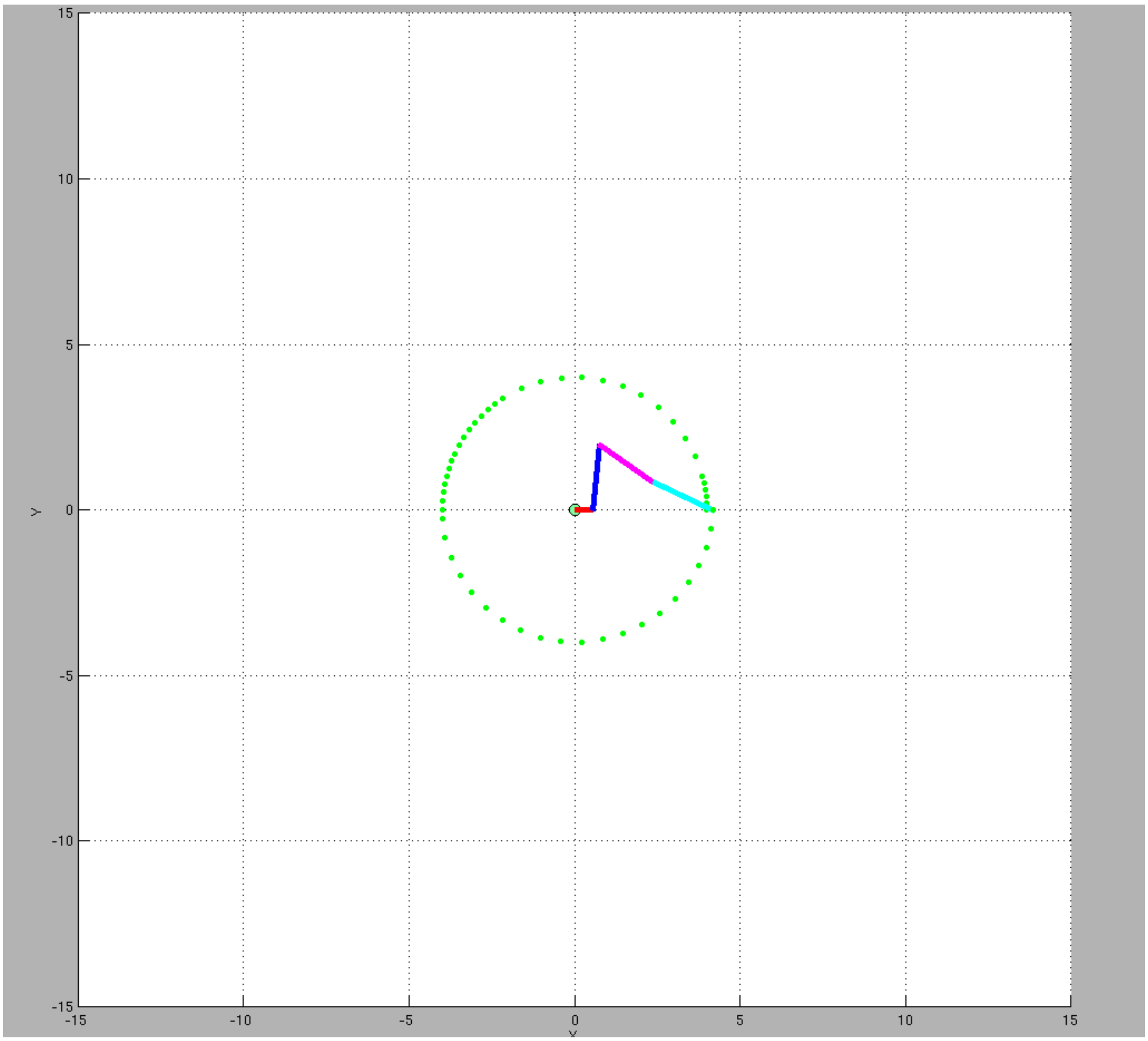


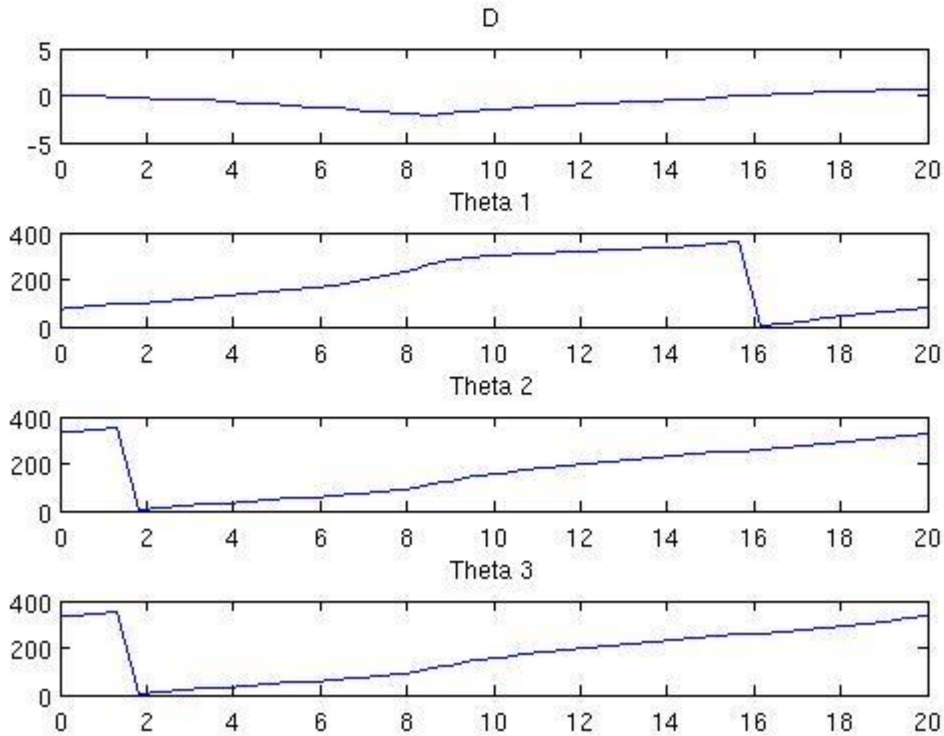
Timeline for $(D, \theta_1, \theta_2, \theta_3)$

Please see drop box folder for code for closed loop control.

3) Closed Loop Control (Task Space):

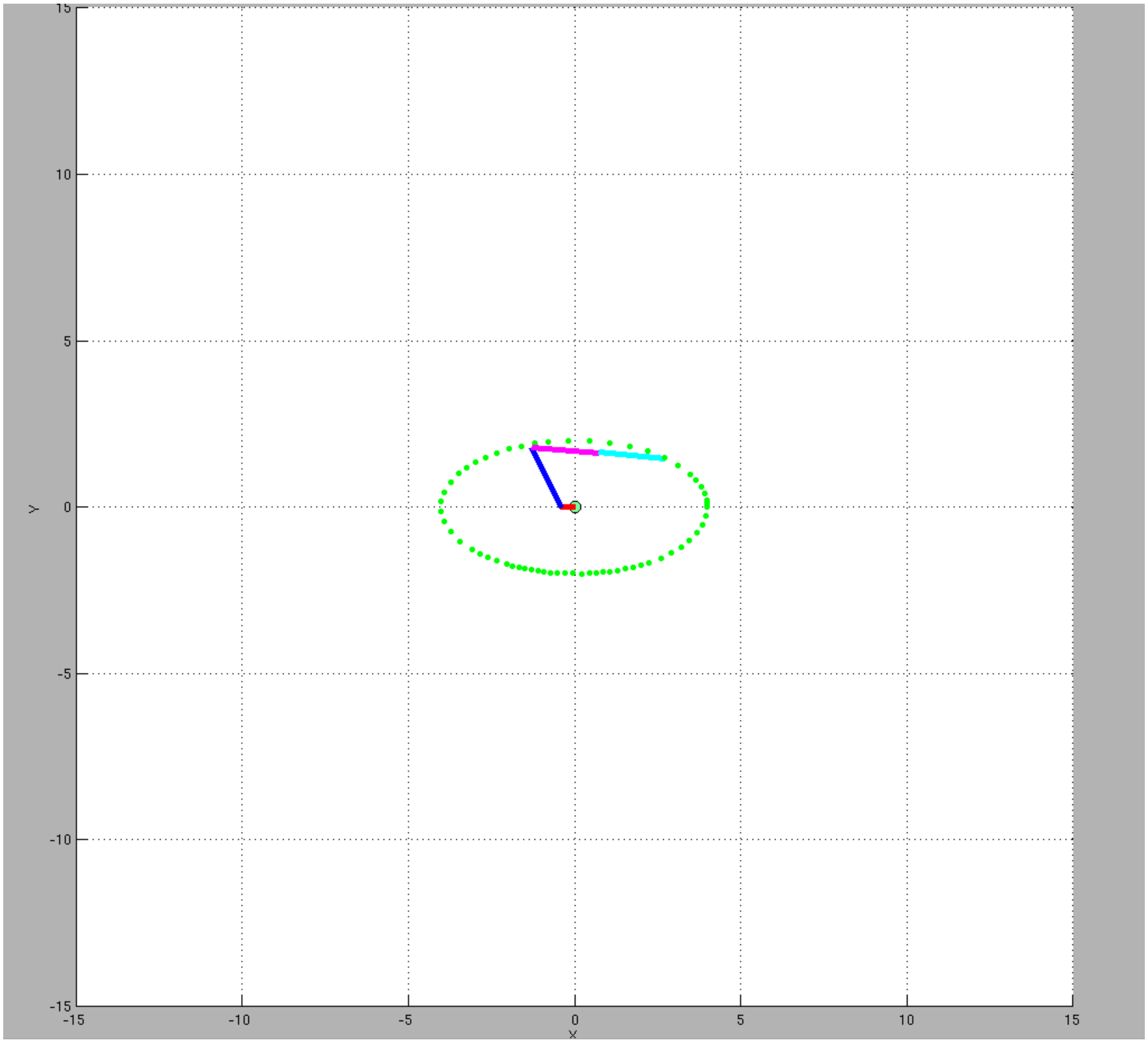
$$\dot{\theta}_{closed} = pinvJ * \left\{ \begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} + K * \left\{ \begin{pmatrix} X_{desired} \\ Y_{desired} \end{pmatrix} - \begin{pmatrix} X \\ Y \end{pmatrix} \right\} \right\}$$





Timeline for $(D, \theta_1, \theta_2, \theta_3)$

Please see drop box folder for code for closed loop control.



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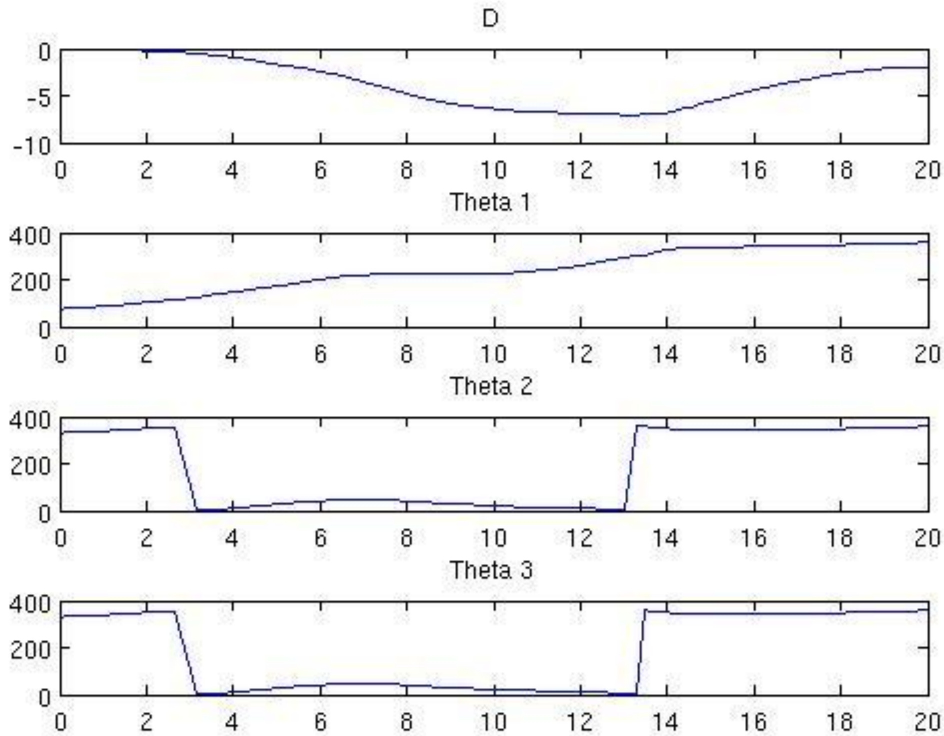


Timeline for $(D, \theta_1, \theta_2, \theta_3)$

Please see drop box folder for code for closed loop control.

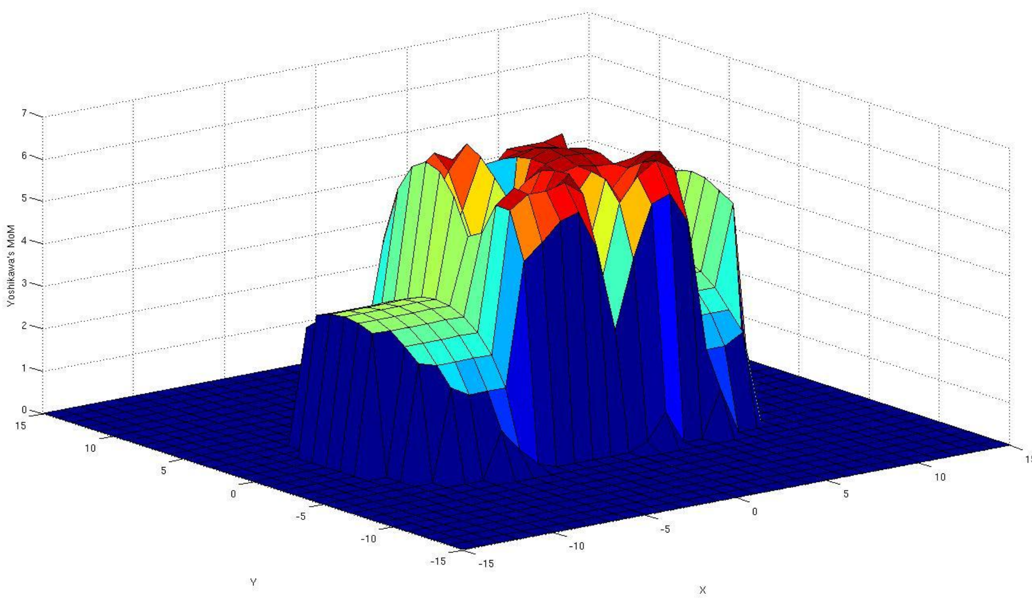
Manipulability:

The $Jacobian\ J$ is given by

$$J = \begin{pmatrix} df_1/_{dD} & df_1/_{d\theta_1} & df_1/_{d\theta_2} & df_1/_{d\theta_3} \\ df_2/_{dD} & df_2/_{d\theta_1} & df_2/_{d\theta_2} & df_2/_{d\theta_3} \end{pmatrix}$$

1) Yoshikawa Measure of Manipulability

$$YMOM = \sqrt[2]{\det(J * transpose(J))}$$

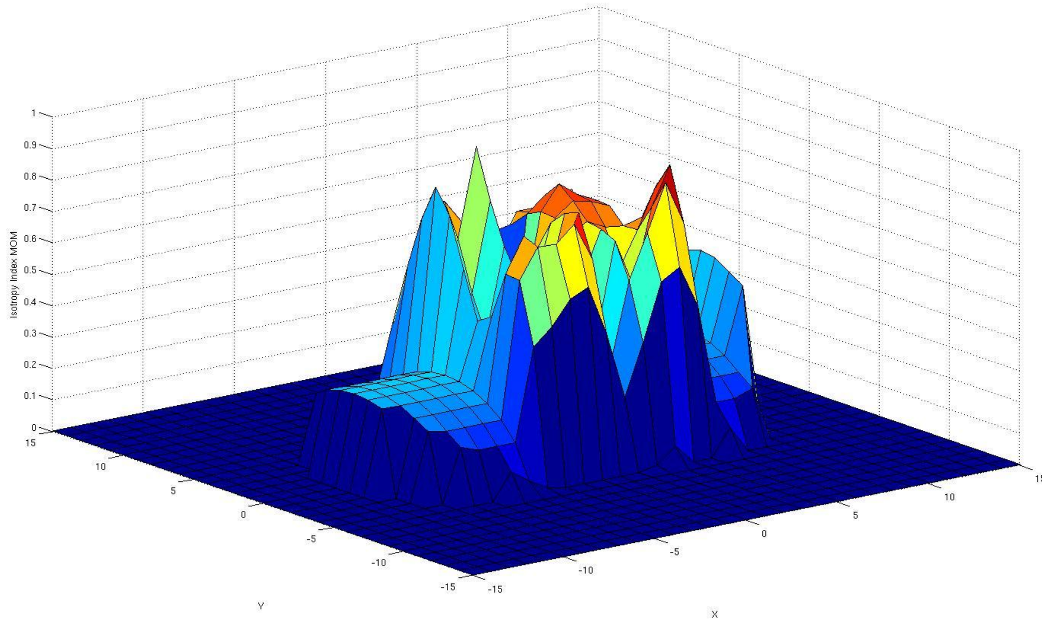


Please see drop box folder for code for Yoshikawa measure of manipulability.

2) Isotropy Index Measure of Manipulability

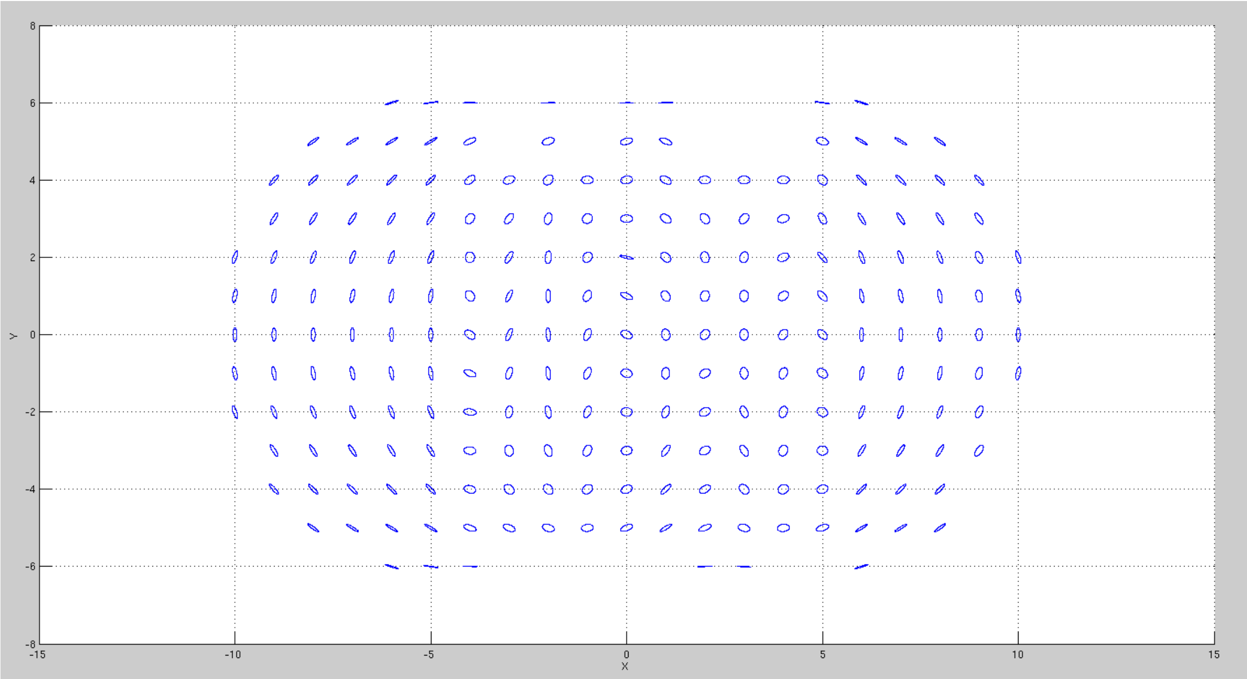
$$U * Sigma * transpose(V) = SVD(J)$$

$$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots \geq \sigma_p$$
$$IIMOM = \sigma_p / \sigma_1$$



Please see drop box folder for code for Isotropy Index measure of manipulability.

3) <u>Manipulability Ellipsoids</u>



Please see drop box folder for code for manipulability ellipsoids.

Appendix

Forward Kinematics http://www.youtube.com/watch?v=yESlf2PsD6g

Inverse Kinematics http://www.youtube.com/watch?v=Bls7HWrkxHA

Workspace http://www.youtube.com/watch?v=f5f1TCPc5Gs

Circle/Ellipse tracing http://www.youtube.com/watch?v=r2oCkZSAfy8

Open Loop Control http://www.youtube.com/watch?v=ylJCD7_EZvU

Task Space Closed Loop Control http://www.youtube.com/watch?v=gfCA28yxF8k

Joint Space Closed Loop Control http://www.youtube.com/watch?v=0BQs7XfaEeA

Isotropy Index Measure of Manipulability http://www.youtube.com/watch?v=tE7zzhBiqyo

Yoshikawa Measure of Manipulability http://www.youtube.com/watch?v=CpM4n1pHX8w

Manipulability Ellipsoids http://www.youtube.com/watch?v=q7aB8xQ6teQ